N64-10400

NASA TECHNICAL NOTE

NASA TN D-2227

NASA TN D-2227

RESEARCH ON PANEL FLUTTER

by D. R. Kobett and E. F. E. Zeydel

Prepared under Contract NASr-63(05) *by*
MIDWEST RESEARCH INSTITUTE
Kansas City, Missouri
for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • NOVEMBER 1963

TECHNICAL NOTE D-2227

RESEARCH ON PANEL FLUTTER

By D. R. Kobett and E. F. E. Zeydel

Prepared under Contract NASr-63(05) by
MIDWEST RESEARCH INSTITUTE
Kansas City, Missouri

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

TABLE OF CONTENTS

List of Figures

TABLE OF CONTENTS (Continued)

List of Figures (Continued)

List of Figures (Continued)

TABLE OF CONTENTS (Concluded)

List of Figures (Concluded)

List of Tables

LIST OF SYMBOLS

a	length of one panel (Fig. 1).
b	width of one panel (Fig. 1).
c_∞	sound velocity in freestream.
E	modulus of elasticity.
g	structural damping coefficient.
h	panel thickness.
j	$\sqrt{-1}$.
k	$\frac{\omega a}{U}$, nondimensional frequency.
K	$\frac{kM}{\beta^2}$.
ℓ	number of panels in the chordwise direction (one in the present study).
$m,\bar{m}$	chordwise modal numbers.
M	$\frac{U}{c_\infty}$, Mach number.
q	integer.
q_m	generalized coordinates associated with the m'th chordwise mode shape.
s	$\frac{a}{b}$, inverse of aspect ratio.
U	free stream velocity.

α	$\dfrac{\rho^2 E \mu^3}{c_\infty^2 \rho_s^3 (1-\nu^2)}$.
β^2	$M^2 - 1$.
μ	$\dfrac{\tau \rho_s}{\rho}$.
ν	Poisson's ratio.
ρ	density in free stream.
ρ_s	panel material density.
σ_x	initial membrane stresses per unit length in chordwise direction.
σ_y	initial membrane stresses per unit length in spanwise direction.
τ	$\dfrac{h}{a}$, thickness ratio.
ω	vibration frequency.

PREFACE

This report covers research initiated by the National Aeronautics and Space Administration, and performed under Contract No. NASr-63(05). The work was administered by Dr. H. Brown of NASA Headquarters, Washington, D. C.

The principal investigator of the program was Dr. E. F. E. Zeydel.

The authors wish to acknowledge the contributions of Mr. D. I. Sommerville who carried out the numerical calculations on the IBM 7090 computer.

D.R.K. and E.F.E.Z.

Approved for:

MIDWEST RESEARCH INSTITUTE

S L Levy

Sheldon L. Levy, Director
Mathematics and Physics Division

10 April 1963

I. INTRODUCTION

It is generally recognized that the panel flutter phenomenon must be taken into account when designing the skin or exterior covering of high speed aircraft and missiles. Usable design criteria based on panel flutter aspects have not as yet been developed, and on the whole the designer is guided by previous experience and/or by experimental data obtained in special tests. It is clear, therefore, that unified design criteria are needed.

Design criteria can be developed from quantitative data that sufficiently account for the effects of all pertinent parameters. At present, quantitative data for the panel flutter phenomenon are not available in such sufficient amounts. This lack of data is due in part to theoretical limitations, in part to the difficulty of performing good experiments, and in part simply to the fact that a large number of pertinent parameters are involved. A theoretical parametric study is therefore conducted here, having as its objective quantitative flutter data for a moderately large range of the important parameters. It is hoped that these data will be useful both in the ultimate development of design criteria and in the intermediate task of evaluating theoretical techniques by means of suitable experiments.

The theoretical parametric study is conducted for an array of simply supported, flat, rectangular panels, extending to infinity in the spanwise direction and to one panel in the chordwise direction. The low supersonic Mach number region is investigated, and the effects of Mach number, aspect ratio, panel material and altitude, are determined. The numerical results are presented in Figs. 2 to 29* in the form of critical flutter boundaries in a generalized parameter plane.

II. EQUATIONS OF MOTION

Flutter equations of motion for an array of simply supported flat panels extending to infinity in the spanwise direction and to a finite number of panels in the chordwise direction are derived in [1]**. In the derivation, small deflection plate theory is used to represent the structural behavior, and since the low supersonic region is of interest, exact, linearized, three-dimensional aerodynamic theory is used to determine the aerodynamic pressure on the plate. An approximate solution to the resulting differential equations

* Figures 1 through 37 appear in Appendix A.

** Numbers in square bracket refer to the bibliography.

of motion for the transverse displacement w is obtained by the Galerkin method. The natural vibration modes of a continuous beam, simply supported at equally spaced intervals, are used to represent the chordwise and spanwise deflection shapes of the panel array. In the spanwise direction only the first mode is retained since it has been shown to yield the most critical flutter requirements. Thus, the transverse displacement of the panel array is approximated by

$$w(x,y,t) = \text{Imag.} \left\{ \sum_{m=1}^{N} q_m \sin \frac{m\pi x}{a} \sin \frac{\pi y}{b} e^{j\omega t} \right\}$$

where the generalized coordinates, q_m , are complex in general. The flutter equations, finally obtained, are*:

$$\frac{(1+jg)}{M^2} \left[(\bar{m}^2+s^2)^2 \frac{\alpha\pi^4}{12}+(\delta_1\bar{m}^2+\delta_2 s^2)\mu\right] q_{\bar{m}} - k^2\mu q_{\bar{m}}$$

$$+ \frac{1}{\beta} \sum_{m=1}^{N} q_m A_{m,\bar{m}} = 0 \qquad (1)$$

where

$$\alpha = \frac{\rho^2 E\mu^3}{c_\infty^2 \rho_s^3 (1-\nu^2)} \qquad \delta_1 = \frac{\pi^2 \sigma_x}{c_\infty^2 \rho_s} \qquad \delta_2 = \frac{\pi^2 \sigma_y}{c_\infty^2 \rho_s} \qquad \mu = \frac{\tau \rho_s}{\rho}$$

$$A_{m,\bar{m}} = j \left[\frac{k(M^2-2)}{\beta^2} + E_m\right] \delta_{m,\bar{m}}$$

$$+ \frac{1}{\ell} \left[(m\pi - F_m)\epsilon_{m,\bar{m}} - (-1)^{\bar{m}\ell} e^{-jKM\ell} G_{m,\bar{m}} + H_{m,\bar{m}}\right]$$

* Equations (1) follow directly from equations (5.7) of [1] if the terms associated with large deflections are dropped and the coefficients regrouped.

$$\delta_{m,\bar{m}} = 0 \text{ for } m \neq \bar{m}$$

$$= 1 \text{ for } m = \bar{m}$$

$$\epsilon_{m,\bar{m}} = 0 \text{ for } (m+\bar{m})\ell = \text{even}$$

$$= \frac{2\bar{m}}{\pi(\bar{m}^2-m^2)} \text{ for } (m+\bar{m})\ell = \text{odd}$$

$$E_m = \sum_{r=1}^{q} (a_r d_{r,m} + b_r f_{r,m})$$

$$F_m = \sum_{r=1}^{q} (a_r e_{r,m} + b_r c_{r,m})$$

$$G_{m,\bar{m}} = \sum_{r=1}^{q} \Big[(a_r g_{r,m,\bar{m}} + b_r h_{r,m,\bar{m}}) \cos \lambda_r \ell$$

$$+ j(a_r h_{r,m,\bar{m}} + b_r g_{r,m,\bar{m}}) \sin \lambda_r \ell\Big]$$

$$H_{m,\bar{m}} = \sum_{r=1}^{q} (a_r g_{r,m,\bar{m}} + b_r h_{r,m,\bar{m}})$$

$$a_r = -\frac{1}{q}\left[(KM-k)^2 + \lambda_r^{\,2}\right]$$

$$b_r = \frac{2k\lambda_r}{q\beta^2}$$

$$c_{r,m} = \frac{KM}{2}\left[\frac{1}{(\lambda_r+m\pi)^2-K^2M^2} - \frac{1}{(\lambda_r-m\pi)^2-K^2M^2}\right]$$

$$d_{r,m} = \frac{KM}{2} \left[\frac{1}{(\lambda_r+m\pi)^2-K^2M^2} + \frac{1}{(\lambda_r-m\pi)^2-K^2M^2} \right]$$

$$e_{r,m} = \frac{1}{2} \left[\frac{\lambda_r+m\pi}{(\lambda_r+m\pi)^2-K^2M^2} - \frac{\lambda_r-m\pi}{(\lambda_r-m\pi)^2-K^2M^2} \right]$$

$$f_{r,m} = \frac{1}{2} \left[\frac{\lambda_r+m\pi}{(\lambda_r+m\pi)^2-K^2M^2} + \frac{\lambda_r-m\pi}{(\lambda_r-m\pi)^2-K^2M^2} \right]$$

$$\lambda_r = \Gamma \cos\left(\frac{2r-1}{4q}\right)\pi$$

$$\Gamma = \sqrt{K^2 + \left(\frac{\pi s}{\beta}\right)^2}$$

and additional definitions are given in the list of symbols.

The summation over the index r in the above terms results from an approximation introduced in the aerodynamic pressure term; namely, Bessel functions are approximated by a sum of circular functions [2]. The argument of the Bessel functions contains the chordwise coordinate x, and consequently the required number of terms, q, depends on the number of chordwise panels in the array. In the present computations for one chordwise panel, satisfactory accuracy is obtained using $q = 6$.

III. FLUTTER BOUNDARIES AND METHOD OF COMPUTATION

The flutter equations (1) are simultaneous algebraic equations for the generalized coordinates q_m. When N chordwise modes are used to represent the deflection function, N equations are obtained, each equation being associated with a particular value of $\bar{m}$. A nontrivial solution of the equations marks a condition of sustained harmonic oscillation or neutral stability for the panel array. Loci of such points plotted in a suitable parametric plane separate regions where flutter will occur from regions of no flutter, i.e., these loci may be interpreted as flutter boundaries. The numerical results of the present study are presented in the form of such flutter boundaries.

Before considering computation details it is useful to look briefly at the general form of the flutter equations. For convenience, the equations are first written in matrix form,

$$\left\{ \begin{bmatrix} P_{m,\bar{m}} \end{bmatrix} \right\} \left\{ \begin{bmatrix} q_m \end{bmatrix} \right\} = 0 \qquad (2)$$

where

$$P_{m,\bar{m}} = \frac{(1+jg)}{M^2} \left[(\bar{m}^2+s^2)^2 \frac{\alpha\pi^4}{12} + (\delta_1\bar{m}^2+\delta_2 s^2)\mu \right] \delta_{m,\bar{m}} - k^2\mu\delta_{m,\bar{m}} + \frac{A_{m,\bar{m}}}{\beta} \qquad (3)$$

Nine basic parameters appear in the equations, namely,

- M - Mach number
- k - reduced frequency
- s - inverse of aspect ratio
- g - structural damping coefficient
- ρ - density in the free stream
- ρ_s - material density
- c_∞ - sound velocity in the free stream
- E - modulus of elasticity
- ν - Poisson's ratio

The $A_{m,\bar{m}}$ are functions of M , k , and s . It is pertinent that the $A_{m,\bar{m}}$ are independent of the parameters associated with panel material and altitude, i.e., ρ , ρ_s , c_∞ , E , and ν . If δ_1 and δ_2 are momentarily ignored $(\sigma_x = \sigma_y = 0)$, it is seen that the material-altitude parameters appear only in α and μ . This means, in essence, that the nine basic parameters listed above, are consolidated into six fundamental groups, M , k , s , g , α , and μ . Thus, flutter boundaries can be obtained in general in terms of α and μ and it is not necessary to make separate computations for each combination of material and altitude of interest. This advantage can be extended to cases of finite σ_x and σ_y by computing for an array of values δ_1 and δ_2, and then interpreting the results in terms of σ_x and σ_y for various values of $\rho_s c_\infty^2$. The procedure is somewhat indirect but the disadvantage is not serious.

Returning now to the details of computation it is recalled that flutter boundaries are to be obtained, consisting of loci of points representing nontrivial solutions of the flutter equations (2). For (2) to have a nontrivial solution it is necessary that

$$\text{Det.}\ \left\{P_{m,\bar{m}}\right\} = 0 \tag{4}$$

which is commonly called the flutter condition. The problem, then, is to determine those combinations of the parameters which satisfy the flutter condition.

Since the determinant of (4) is complex and only real values of the parameters have physical significance, two parameters must be kept free in order to satisfy (4). It is convenient to take α and μ as the free parameters. Flutter boundaries in the α-μ plane are then obtained for a selected set of values M, s, g, δ_1, δ_2 by determining the α-μ pairs which satisfy (4) for various judiciously chosen values of reduced frequency k. Two procedures were used for finding these α-μ pairs. The first procedure to be described was used initially and then discarded in favor of the second for reasons that will be pointed out.

The first method consists of a trial and error procedure for determining the α-μ pairs which cause both the real and imaginary parts of the determinant to vanish simultaneously. For a particular value of α and a number of values of μ the real part, R, and the imaginary part, I, of the flutter determinant are calculated. The values of μ for which $R = 0$ (R_o) and $I = 0$ (I_o) are obtained by interpolation. Next, α is varied and the process repeated. In this manner the functions $R_o(\alpha)$ and $I_o(\alpha)$ become known. The intersections of $R_o(\alpha)$ and $I_o(\alpha)$ determine the flutter points and are obtained by interpolation. Although this method yields satisfactory results, it leaves something to be desired in the way of economy and convenience of application. The principal difficulty arises from the fact that the functions $R_o(\alpha)$ and $I_o(\alpha)$ are multi-valued and composed of several branches in the α-μ plane.

The second method for determining α and μ is as follows. The determinant of (4) can be written as a polynomial in μ with complex coefficients that depend on α,

$$D(\mu) = a_o\mu^m + a_1\mu^{m-1} + \ldots\ldots + a_m$$

Because D should vanish and μ must be real, the flutter condition becomes

$$f(\mu) = \mu^m + (a_1/a_o)_R\mu^{m-1} + \ldots\ldots + (a_m/a_o)_R = 0$$

$$g(\mu) = (a_1/a_o)_I\mu^{m-1} + \ldots\ldots\ldots + (a_m/a_o)_I = 0$$

where the R and I subscripts mean "real part of" and "imaginary part of," respectively. Thus, at flutter, $f(\mu)$ and $g(\mu)$ must have at least one common root. The condition that two polynomials in one variable have a common root can be obtained from Euclid's algorithm [3], as follows. Since f is of higher degree in μ, it can be divided by g to give

$$f(\mu) = Q_o(\mu)g(\mu) + R_1(\mu)$$

where Q_o is the quotient polynomial and R_1 is the remainder polynomial. Continuing the process by dividing g by R_1, etc., the following system of identities results.

$$g(\mu) = Q_1(\mu)R_1(\mu) + R_2(\mu)$$

$$R_1(\mu) = Q_2(\mu)R_2(\mu) + R_3(\mu)$$

$$\cdots\cdots\cdots\cdots\cdots$$

$$R_{p-1}(\mu) = Q_p(\mu)R_p(\mu) + R_{p+1}$$

Now g, R_1, R_2, ... are polynomials of decreasing degrees in μ so that after a finite number of steps a remainder, R_{p+1}, is reached which is independent of μ. It can be shown that the condition that f and g have at least one common root is $R_{p+1} = 0$. Further, the common roots can be obtained from $R_p(\mu) = 0$. Since a_o, a_1, ..., a_m depends on α, R_{p+1} is a function of α and the flutter condition is $R_{p+1}(\alpha) = 0$. The values of α corresponding to flutter can thus be determined by the usual interpolation procedures. The associated values of μ follow from $R_p(\mu) = 0$.

This second method for determining α and μ is more straightforward than the first because only one parameter, α, is varied and μ follows as an interim result of the procedure. Also pertinent is the fact that the function $R_{p+1}(\alpha)$, which must be monitored in the computer program, is single-valued in contrast with the multi-valued functions $R_o(\alpha)$ and $I_o(\alpha)$ of the first method. Most of the numerical results were obtained using this second method.

Computation of the flutter boundaries are carried out on the IBM 7090 computer. In all cases the chordwise deflection shape of the panel array is approximated by the first four natural vibration modes of a simply supported beam. As noted earlier, the spanwise deflection is approximated by the first mode of a continuous beam, simply supported at equally spaced intervals. The scope of the parametric study is discussed in the following section.

IV. SCOPE AND RESULTS OF PARAMETRIC STUDY

Flutter characteristics are computed for semi-infinite panel arrays of the type shown in Fig. 1. The desire of the parametric study is to examine the effects of Mach number, length/width ratio, panel material, altitude, and initial membrane forces. Computations are therefore carried out for length/width ratios, s, of 1/4, 1/2, 1, 2, and 4 at each of six Mach numbers, 1.1, 1.25, 1.35, $\sqrt{2}$, 1.5, and 2.0.* Generalized parameters are employed so that the results are applicable to a wide variety of panel materials and altitudes. At the least, the altitude range sea-level to 50,000 ft. is covered for brass, steel, titanium, aluminum, glass, magnesium, and copper. (The material-altitude coverage is described more precisely later.) The effects of membrane forces are not examined because of time limitations; all results are obtained for zero membrane forces ($\sigma_x = \sigma_y = 0$).

* Two cases, $s = 4$ and $M = 1.1$ and 1.25 were not completed because of numerical difficulties.

The numerical results of the parametric study are presented in Figs. 2 through 29 and Tables I through V.* Before examining these results in detail, it is useful to first consider the parameters used for displaying the flutter boundaries. It may be recalled that, earlier, two general parameter groupings α and μ were identified, which consolidate the specific quantities associated with panel material and altitude. Use is made here of α and μ to facilitate concise presentation of the numerical results. In the figures, the parameter on the ordinate is $\alpha^{1/3} = \tau[E/\rho c_\infty{}^2(1-\nu^2]^{1/3}$. On the abscissa the parameter is $(\mu^3/\alpha)^{1/2} = [c_\infty\rho_s/\rho][\rho_s(1-\nu^2/E]^{1/2}$ which form was selected to eliminate the thickness ratio τ . The two parameters are made up of the thickness ratio τ , material constants ρ_s , E , and ν , and airstream properties ρ and c_∞ . Therefore, the computed stability boundaries can be applied conveniently, either to particular material-altitude combinations or to combinations of material and wind tunnel conditions ρ and c_∞ . The extent of coverage provided by the parametric study is indicated in Fig. 36 where $(\mu^3/\alpha)^{1/2}$ is plotted against altitude for some typical panel materials.

The ordinate parameter $\tau[E/\rho c_\infty{}^2(1-\nu^2)]^{1/3}$ is closely related to a dynamic pressure parameter, $\tau[\beta E/q]^{1/3}$, which is commonly used in the literature for correlation of experimental data. The two parameters are related by,

$$\tau[E/\rho c_\infty{}^2(1-\nu^2)]^{1/3} \times [2\beta(1-\nu^2)/M^2]^{1/3} = \tau[\beta E/q]^{1/3}$$

Returning now to consideration of the numerical results, it is reiterated that the results are completely contained in Figs. 2 through 29, where the computed flutter boundaries are shown in detail, one boundary to each figure.** In subsequent figures the boundaries are reproduced together, in lesser detail, for purposes of comparison. The region above the boundary is the stable region as indicated. The stability boundaries, therefore, define the minimum thickness that will prevent flutter. A somewhat arbitrary terminology is used to indicate the form of the flutter vector. A "first mode dominant" boundary, for instance, is one for which the flutter vector consists predominantly of the first generalized coordinate, q_1 , the magnitude of the second largest being less than $q_1/2$. A "first and second mode coupled" boundary, on the other hand, is one where q_1 is largest and q_2 second largest, with $q_2 > q_1/2$.

---

* Tables I through V are shown in Appendix B.

** The numerical data from which the boundaries are constructed are given in Tables I to V. In the tables, the generalized coordinates, q_m , are given in normalized polar form, referenced to the magnitude and phase of the largest coordinate.

Two interesting features are seen in the detailed stability boundaries (Figs. 2 to 29). First of all, there are cases where the critical flutter vector changes abruptly from one mode of dominance to another. For instance, for $M = \sqrt{2}$ and $s = 1/4$ (Fig. 5), an aluminum panel will flutter most readily in the third mode at sea level, the fourth mode at higher altitudes, and finally in the first and second modes coupled at altitudes above approximately 17,000 ft. Brass, steel, or titanium panels on the other hand, will always flutter in the coupled mode. This switching of flutter modes, which is noted for all length-width ratios except $s = 4$, is accompanied by significant changes in flutter frequency (see the tables).

The second feature of interest is that the stability boundaries are only fairly flat in the high Mach number regime, i.e., the parameter $\tau\left[E/\rho c_\infty{}^2(1-\nu^2)\right]^{1/3}$ at flutter is insensitive to changes in panel material and altitude only for $M > \sqrt{2}$, as theoretically expected. This point is clearly illustrated in Figs. 30 through 34.

In Figs. 30 to 34 the stability boundaries for particular values of length-width ratio are superposed. The uppermost boundaries are critical for the Mach number range 1.1 to 2.0 considered in the parametric study. It is seen that the lower Mach numbers are critical for small s and that the critical Mach number becomes larger with increasing s. The flutter vectors associated with the critical boundaries are as follows.

$s = 1/4$; first mode dominant

$s = 1/2$; second mode dominant

$s = 1$; third mode dominant

$s = 2$; fourth mode dominant

$s = 4$; third and fourth mode coupled

Thus, as s increases, the higher modes and higher Mach numbers become increasingly important. Also, examination of the tabulated data shows that, in general, coupling between modes increases as s increases.

In Fig. 35 the uppermost boundaries from Figs. 30 to 34 are shown in superposition, to illustrate the pronounced stabilizing effect associated with increasing values of the length-width ratio s. The effect is greatest at the smaller s values where the lower modes dominate, but the reduction in thickness required to prevent flutter is still appreciable in going from $s = 2$ to $s = 4$.

V. CONCLUSIONS AND RECOMMENDATIONS

The stability boundaries in Figs. 2 through 29 provide information on the flutter characteristics of flat panels under a wide variety of conditions. Detailed information for particular materials, altitudes, or wind tunnel conditions can be obtained conveniently using intermediate relationships of the type shown in Figs. 36 and 37. This detailed information should be of use to the designer in lieu of presently unavailable design criteria, and to the experimentalist in planning experiments and analyzing experimental observations.

A most critical need at this time is valid comparisons between theory and experiment. The data presented here should facilitate such comparisons.

For the future it is suggested first of all, that the length-width ratio $s = 4$, treated here, should be further examined using six or more modes in the analysis. It is felt that the general characteristics cited in the previous section imply that the higher modes may have significant effect. It is further suggested that the effects of initial membrane forces and more than one panel in the chordwise direction be investigated, particularly since the theory is available (Eq.(1)). Finally, a parametric study similar to the one reported here, should be conducted for panels with partially clamped edges since it is expected that in practical application (and experiments) boundary conditions are somewhere between simply supported and clamped.

Midwest Research Institute
Kansas City, Missouri

REFERENCES

1. Zeydel, E. F. E., "Large Deflection Panel Flutter," AFOSR Tech. Note 1952, January, 1962.

2. Luke, Y. L., and A. D. St. John, "Supersonic Panel Flutter," WADC Tech. Report 57-252, July, 1957.

3. Bocher, M., "Introduction to Higher Algebra," The MacMillan Company (1936).

APPENDIX A

FIGURES 1 THROUGH 37

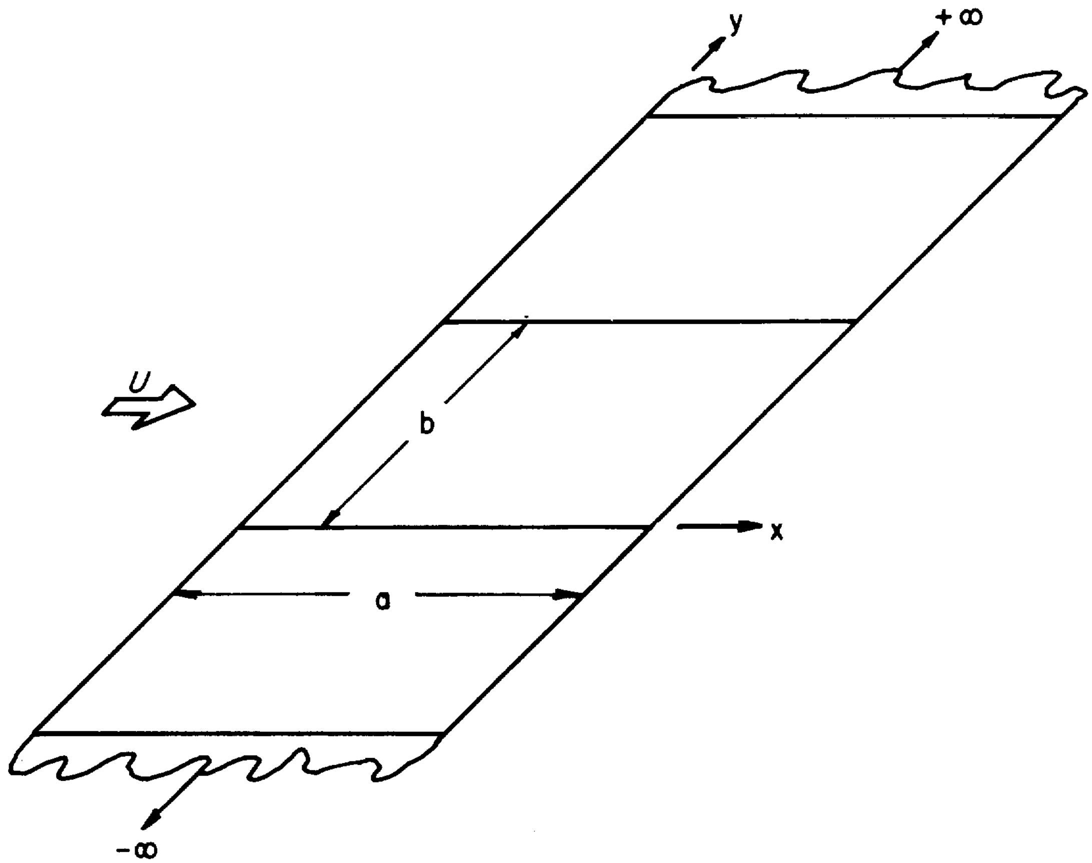

Fig. 1 - Panel Array

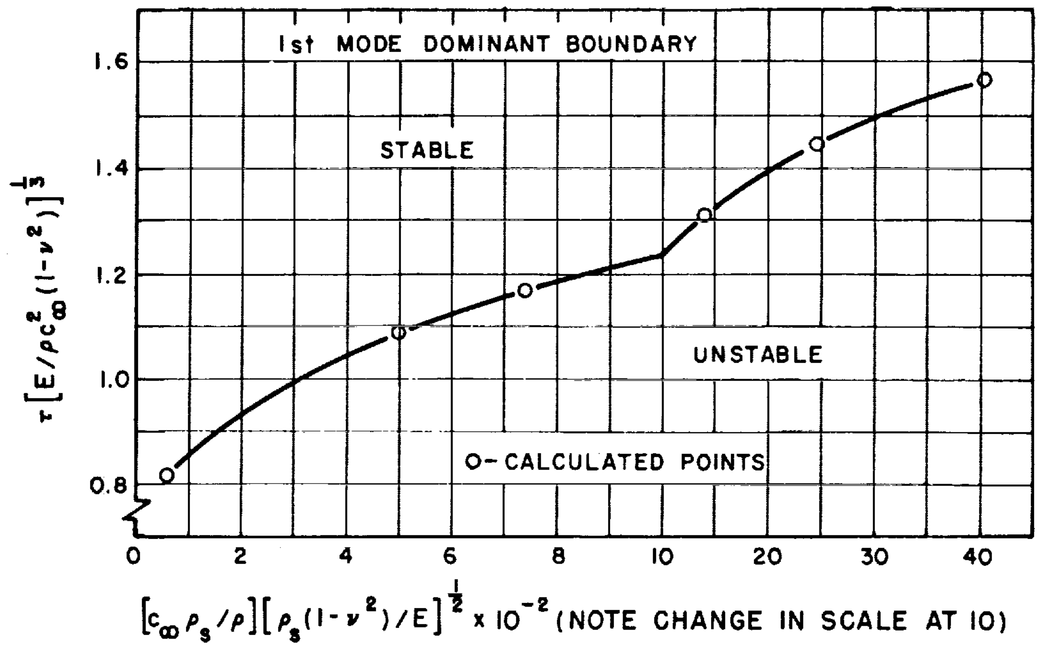

Fig. 2 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.1, s = 1/4, g = 0.01 (Table I)

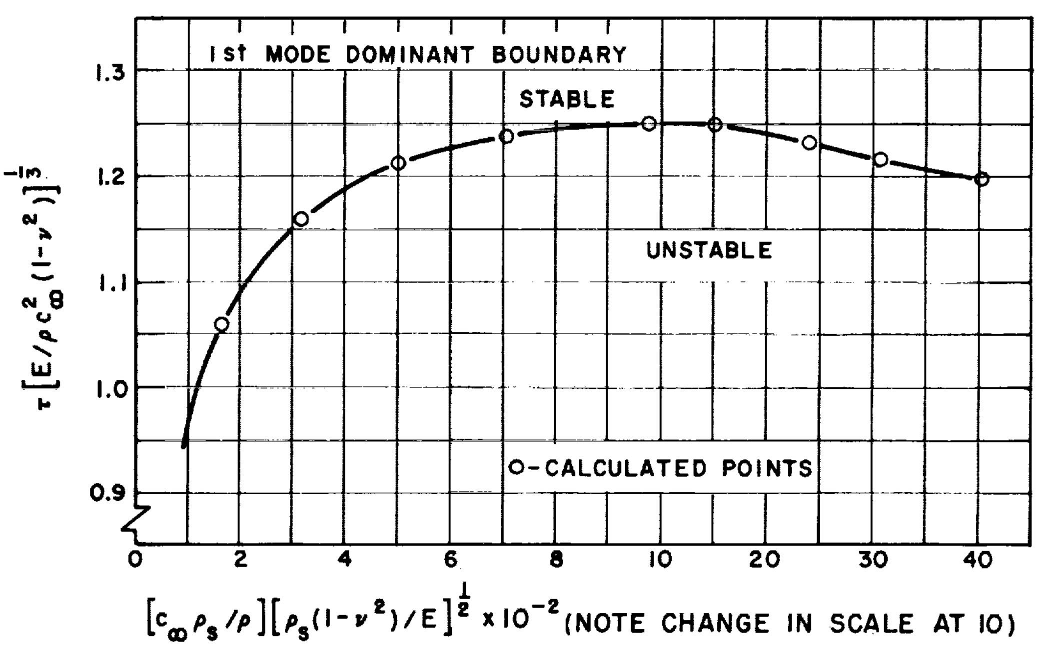

Fig. 3 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.25, s = 1/4, g = 0.01 (Table I)

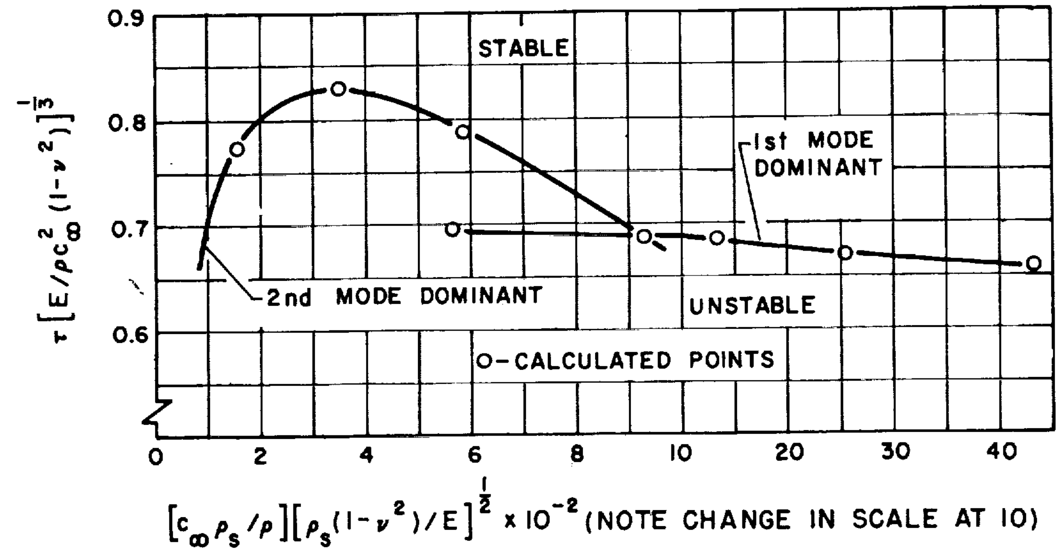

Fig. 4 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, $M = 1.35$, $s = 1/4$, $g = 0.01$ (Table I)

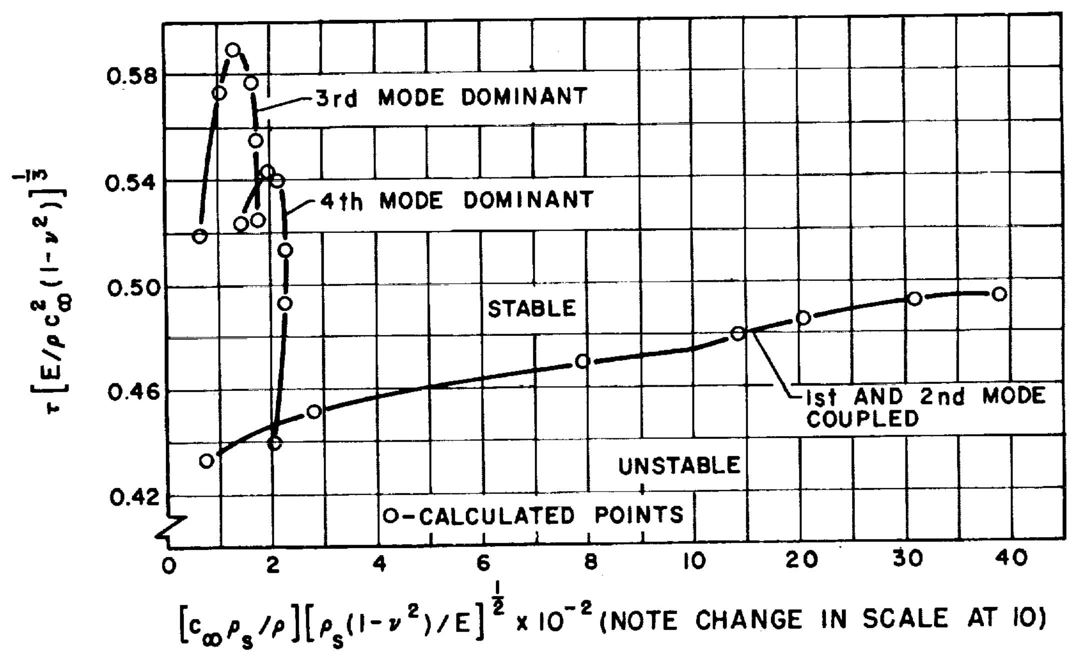

Fig. 5 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, $M = \sqrt{2}$, $s = 1/4$, $g = 0.01$ (Table I)

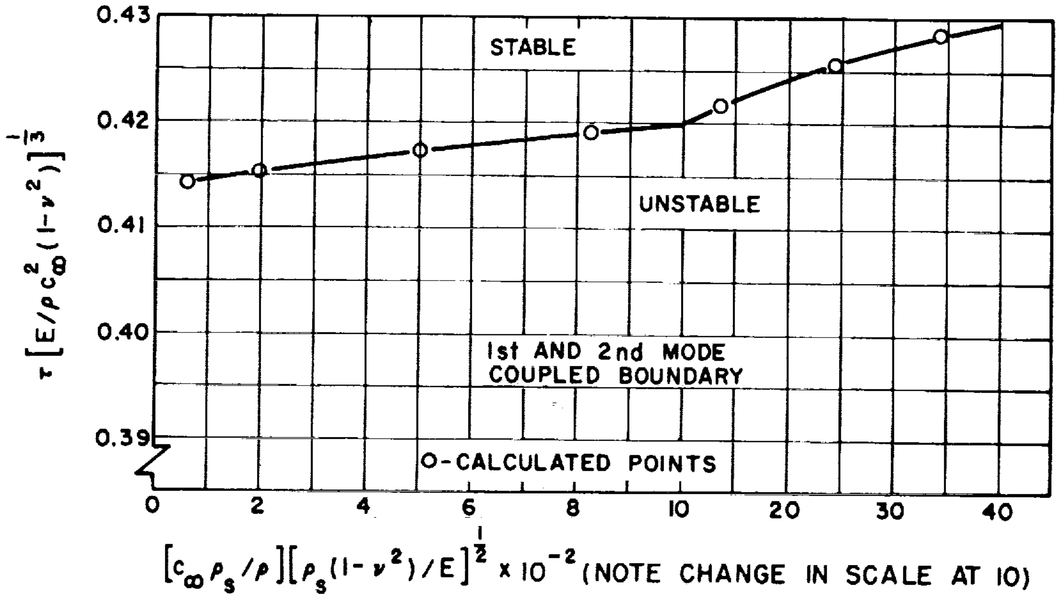

Fig. 6 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.5, s = 1/4, g = 0.01 (Table I)

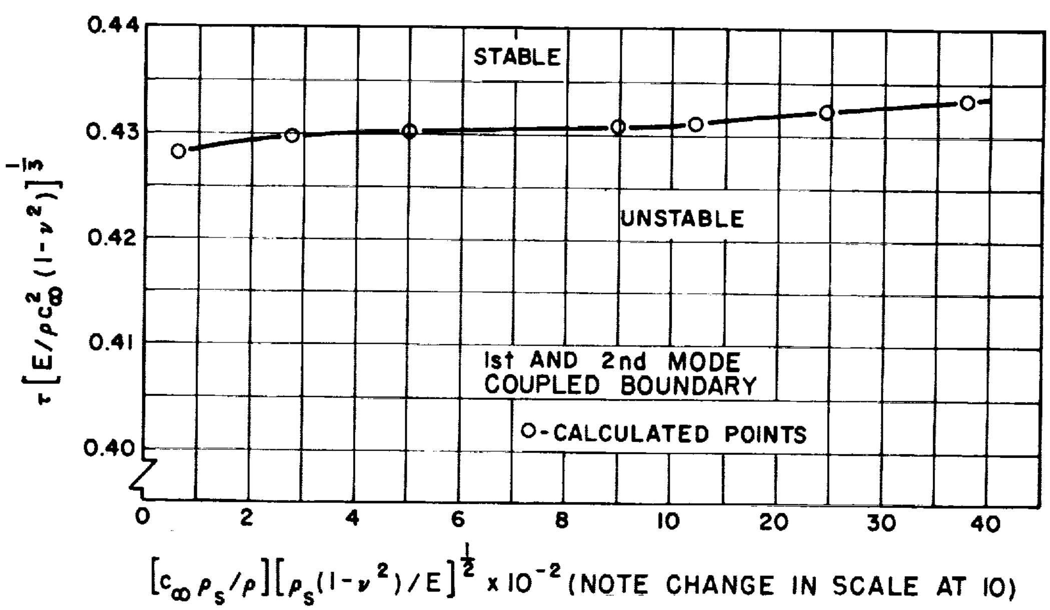

Fig. 7 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 2.0, s = 1/4, g = 0.01 (Table I)

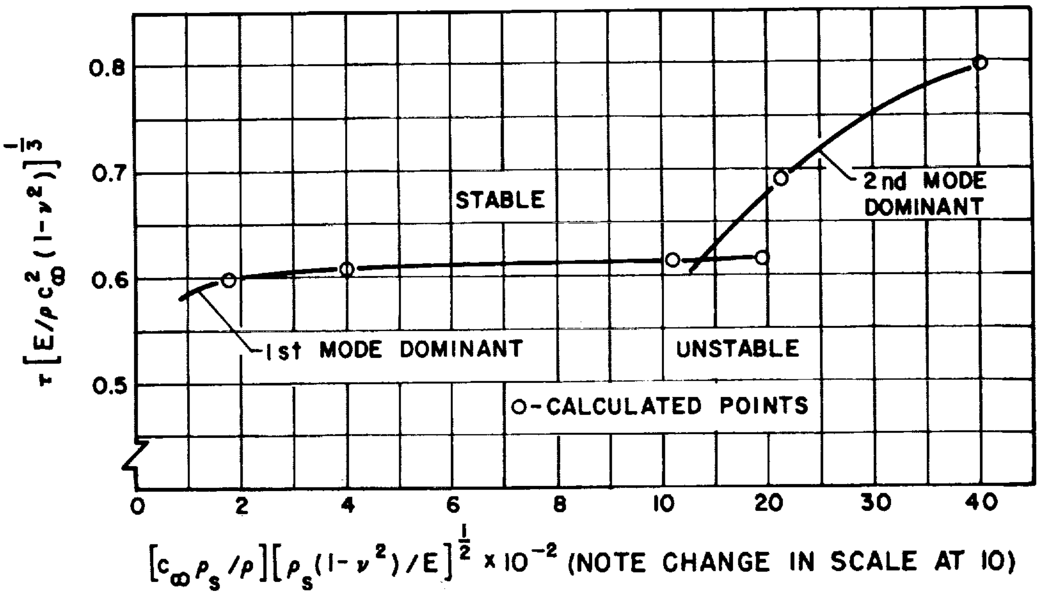

Fig. 8 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.1, s = 1/2, g = 0.01 (Table II)

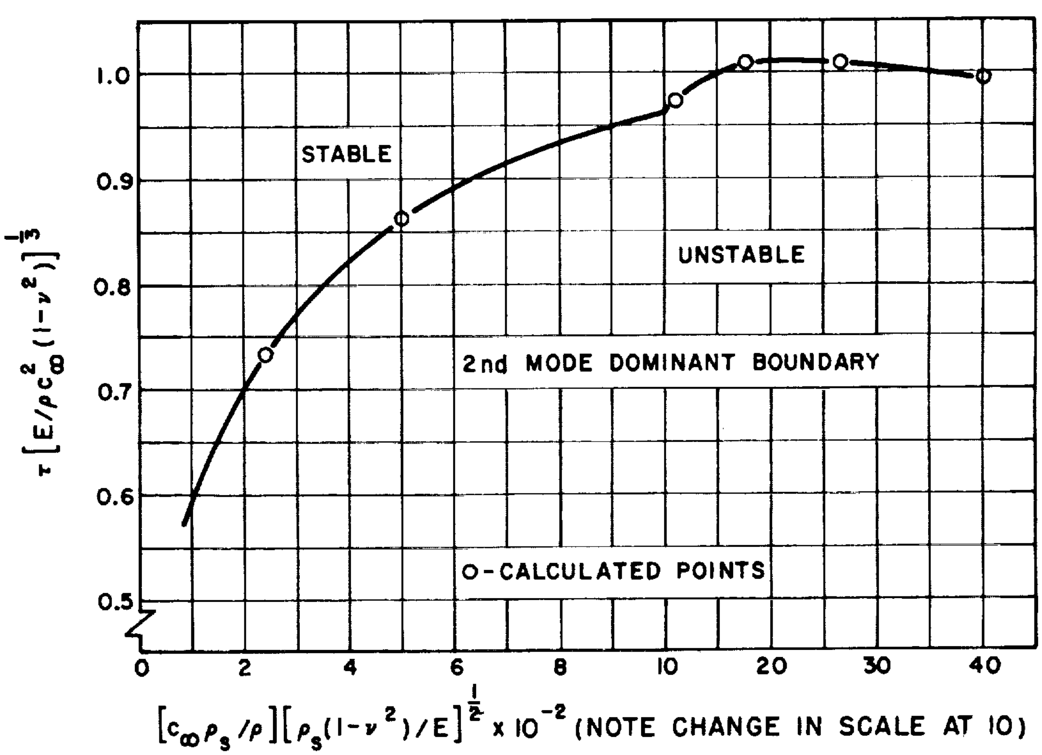

Fig. 9 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.25, s = 1/2, g = 0.01 (Table II)

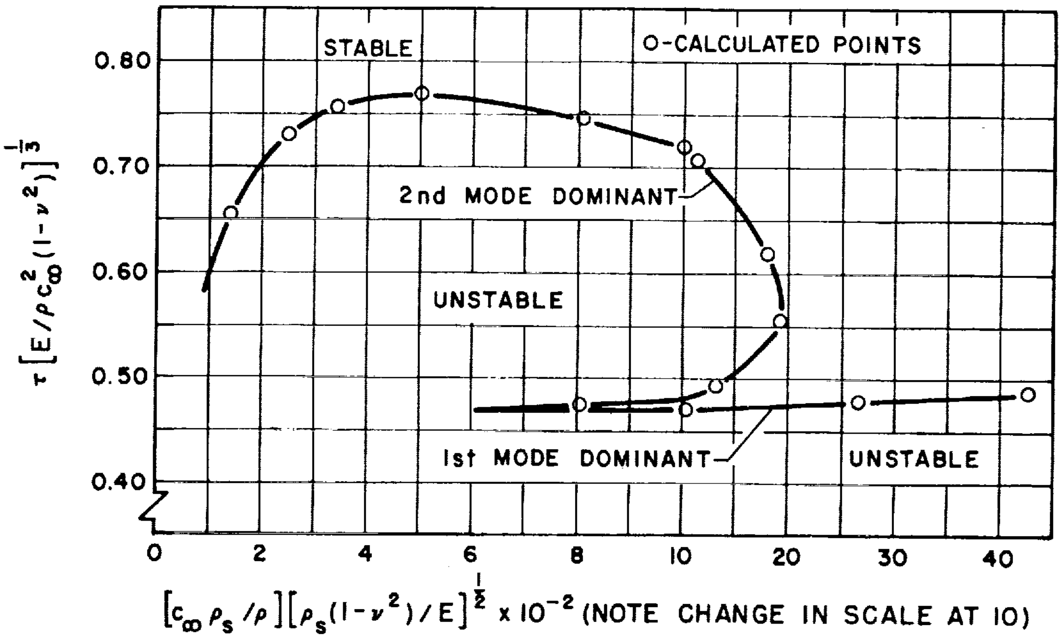

Fig. 10 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.35, s = 1/2, g = 0.01 (Table II)

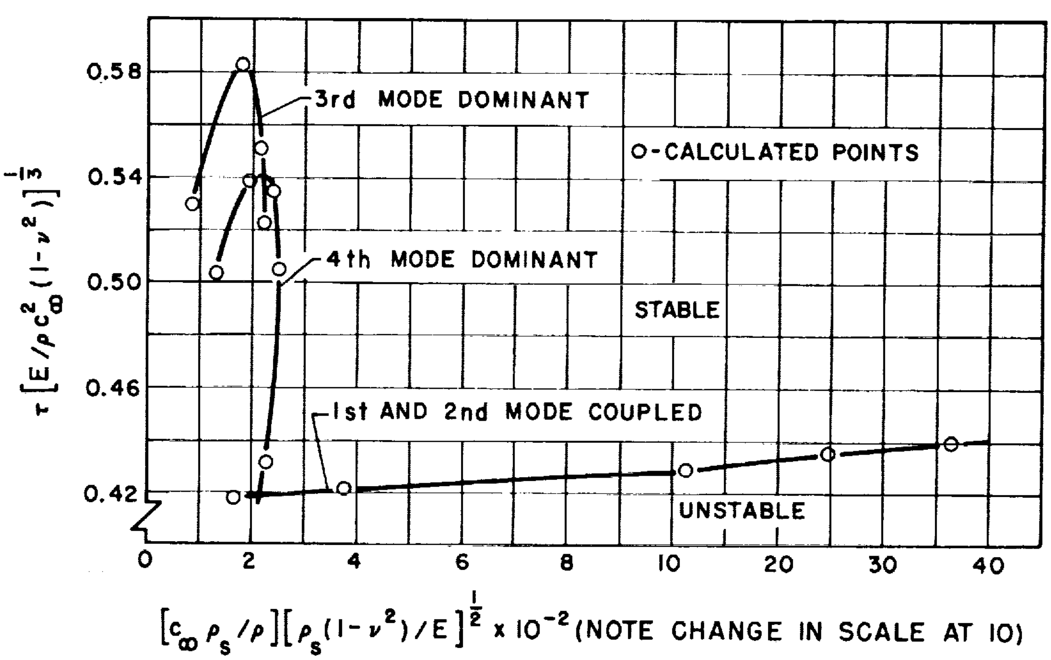

Fig. 11 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = $\sqrt{2}$, s = 1/2, g = 0.01 (Table II)

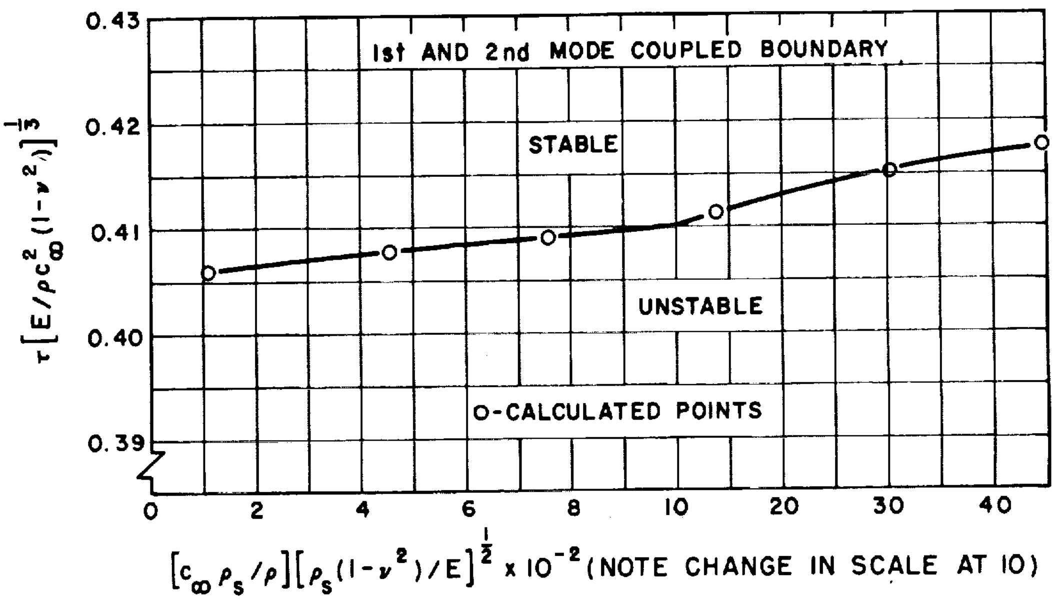

Fig. 12 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.5, s = 1/2, g = 0.01 (Table II)

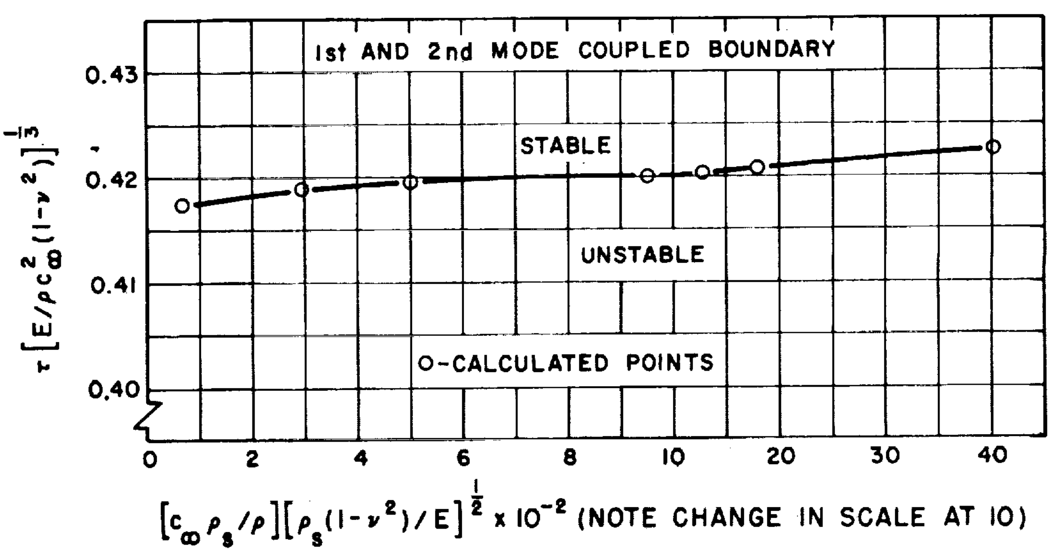

Fig. 13 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 2.0, s = 1/2, g = 0.01 (Table II)

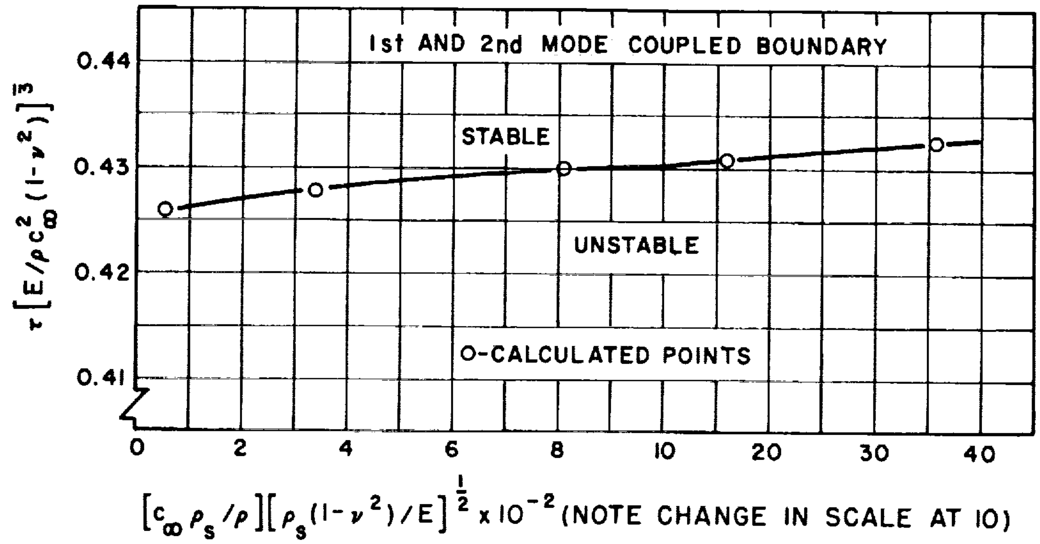

Fig. 14 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.1, s = 1, g = 0.01 (Table III)

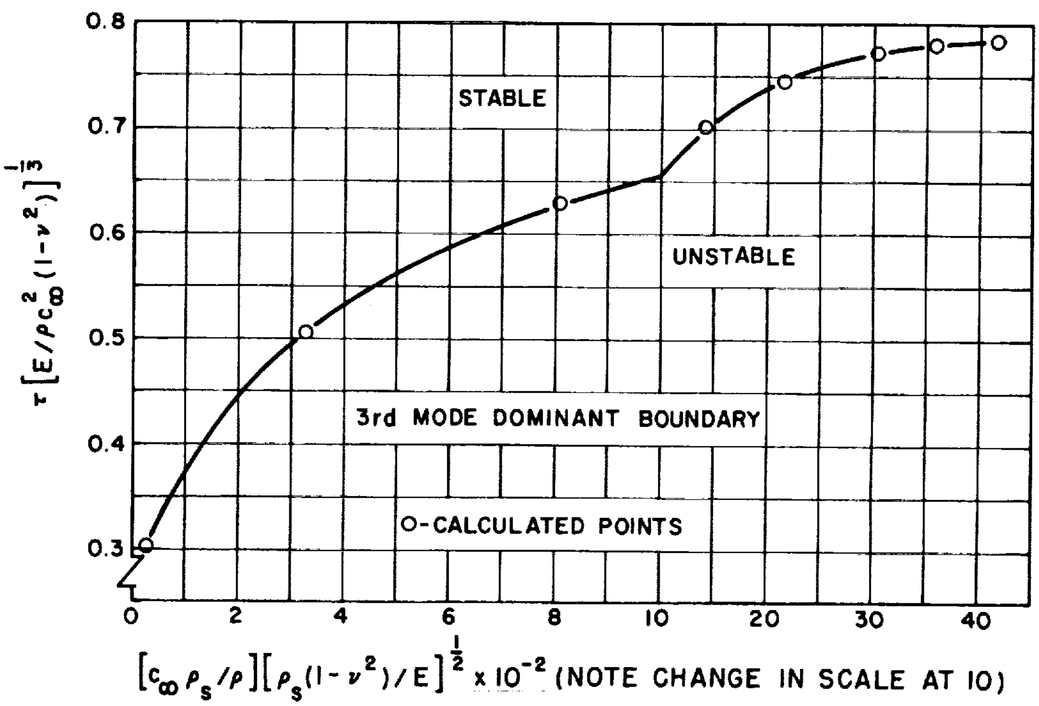

Fig. 15 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.25, s = 1, g = 0.01 (Table III)

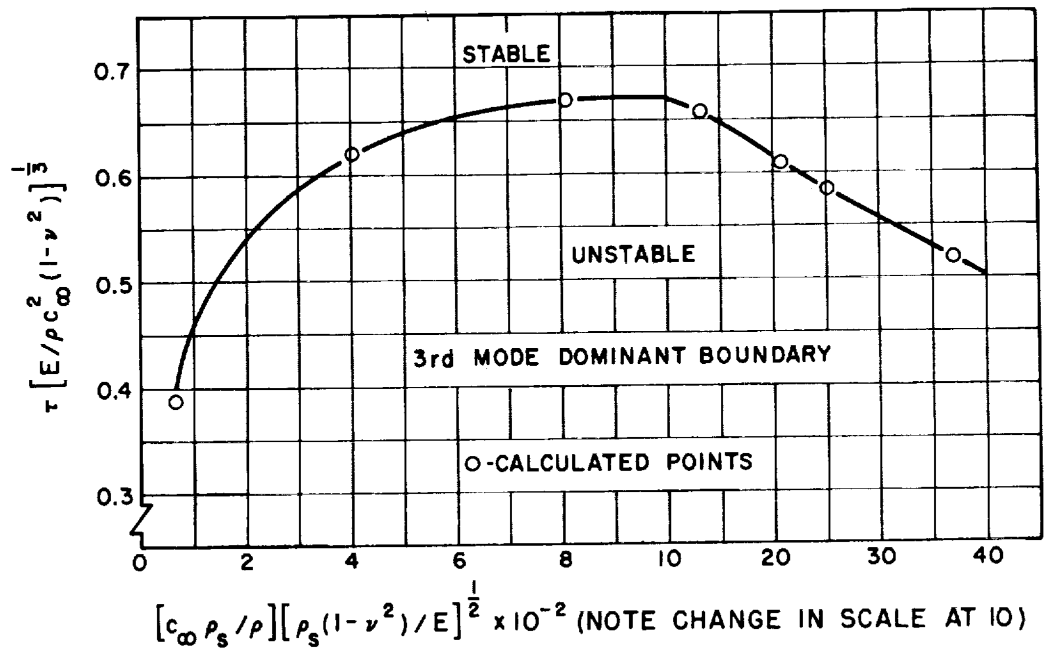

Fig. 16 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.35, s = 1, g = 0.01 (Table III)

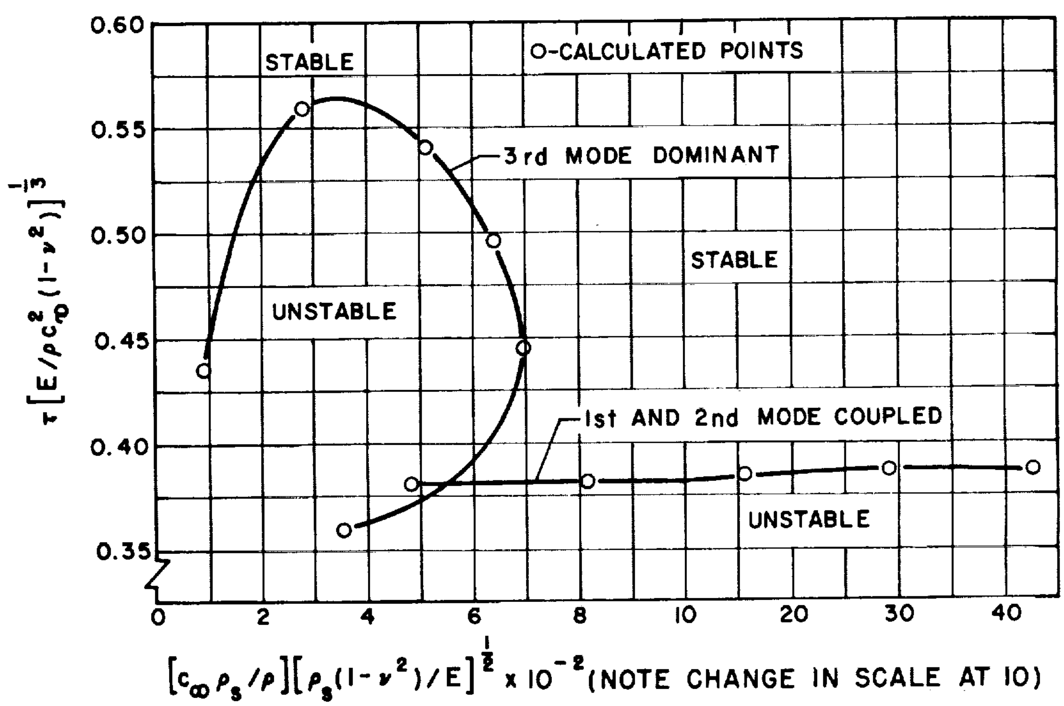

Fig. 17 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = $\sqrt{2}$, s = 1, g = 0.01 (Table III)

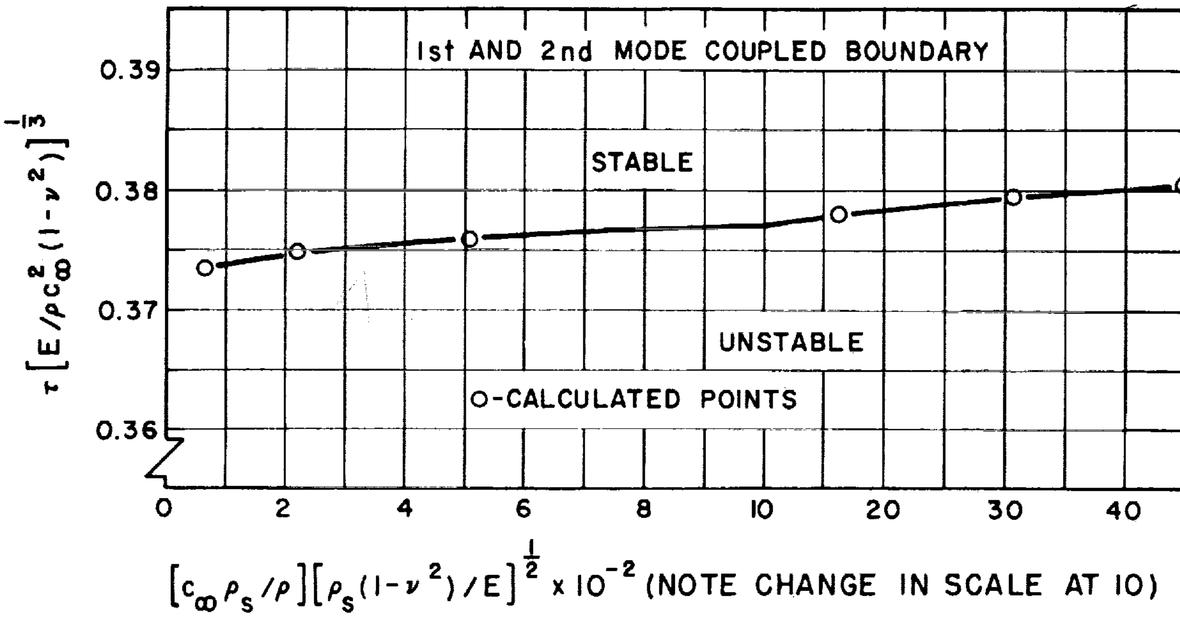

Fig. 18 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.5, s = 1, g = 0.01 (Table III)

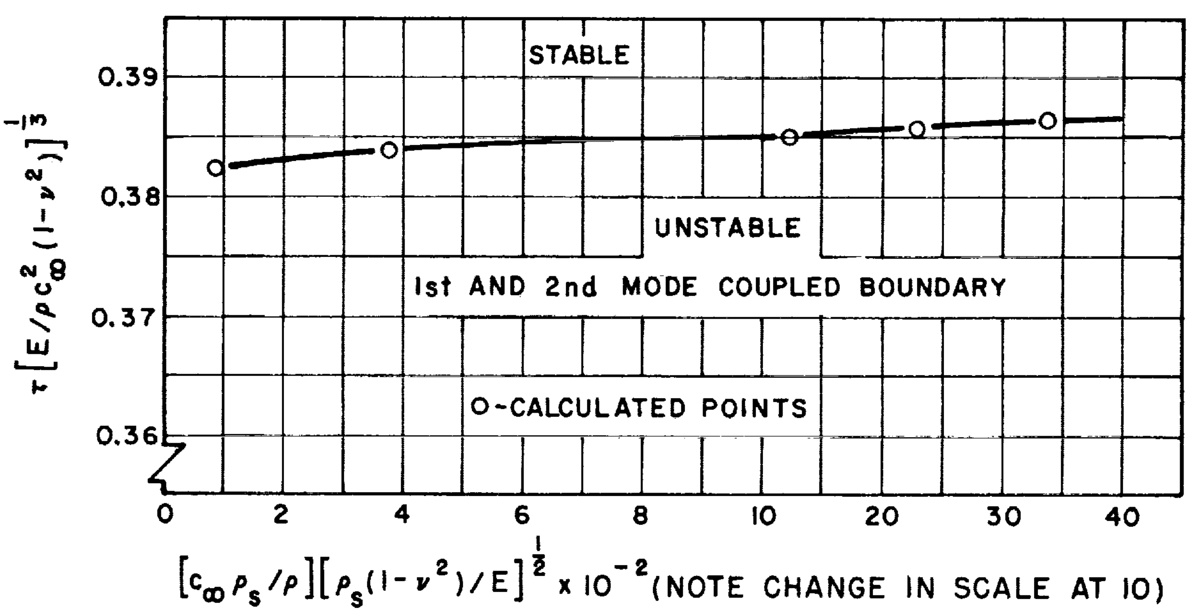

Fig. 19 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 2, s = 1, g = 0.01 (Table III)

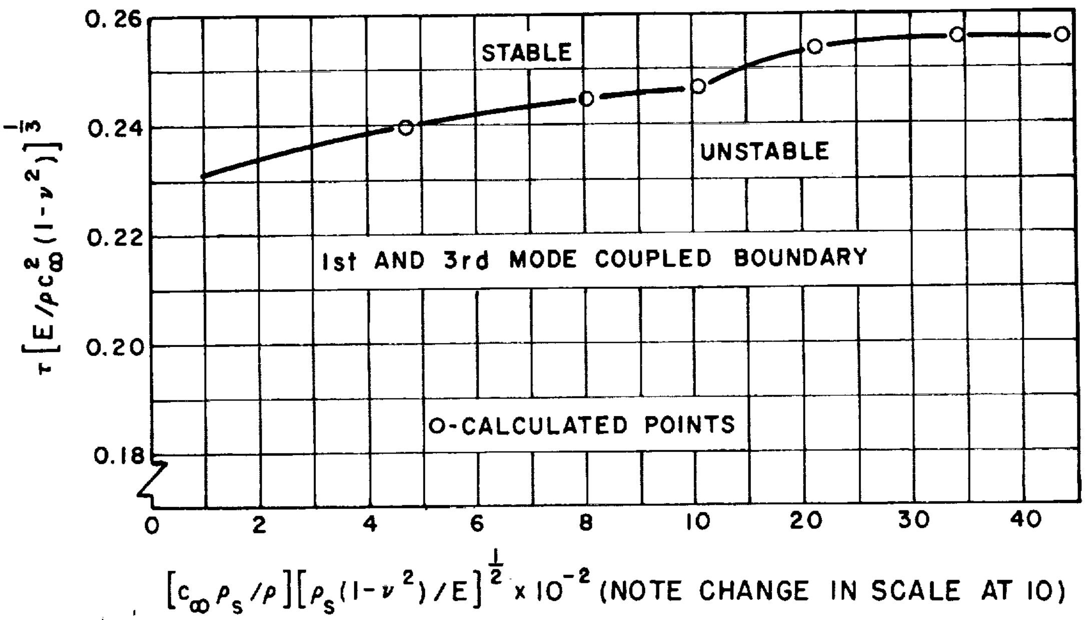

Fig. 20 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.1, s = 2, g = 0.01 (Table IV)

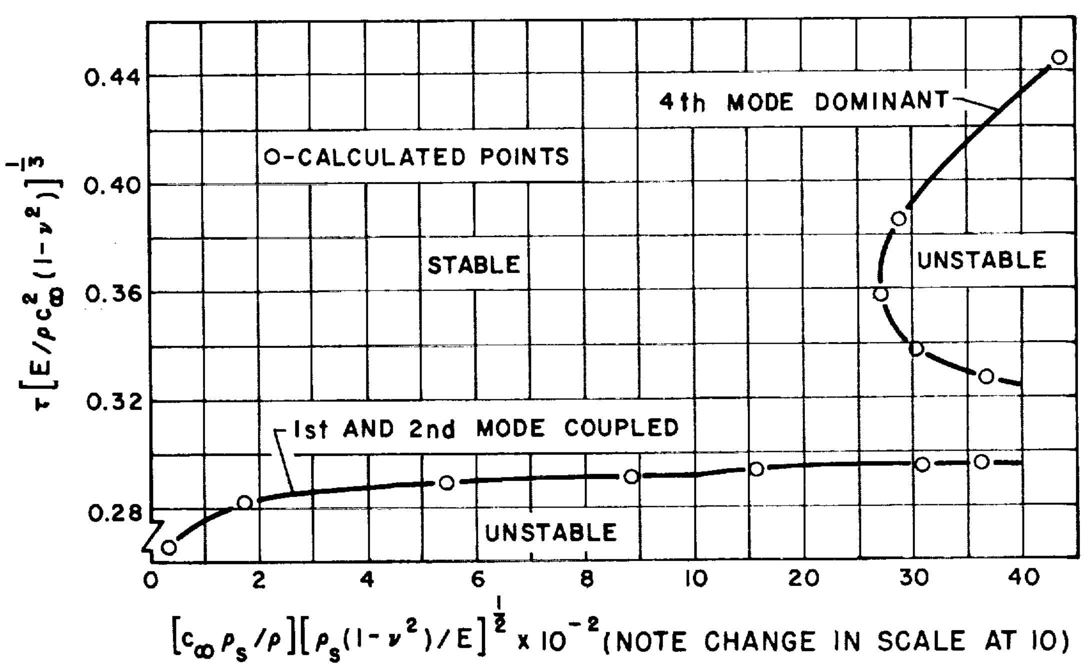

Fig. 21 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.25, s = 2, g = 0.01 (Table IV)

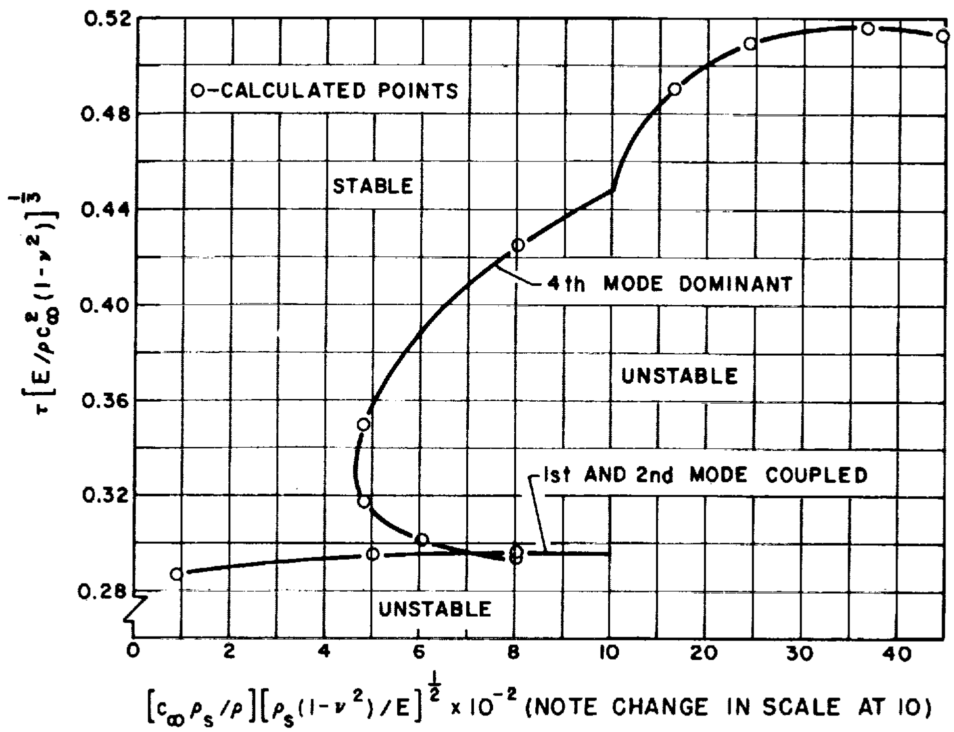

Fig. 22 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.35, s = 2, g = 0.01 (Table IV)

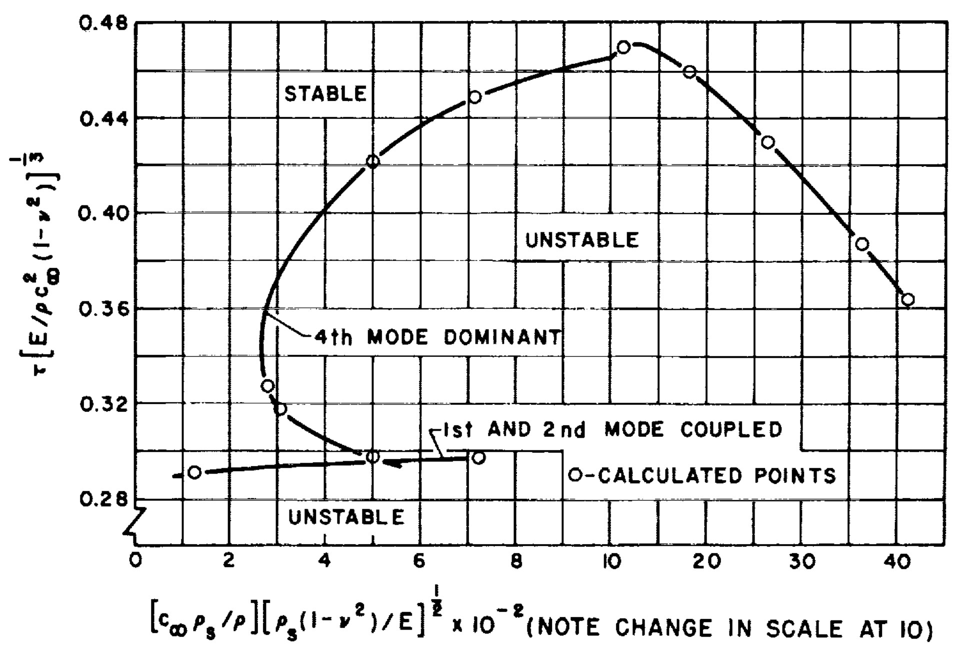

Fig. 23 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = $\sqrt{2}$, s = 2, g = 0.01 (Table IV)

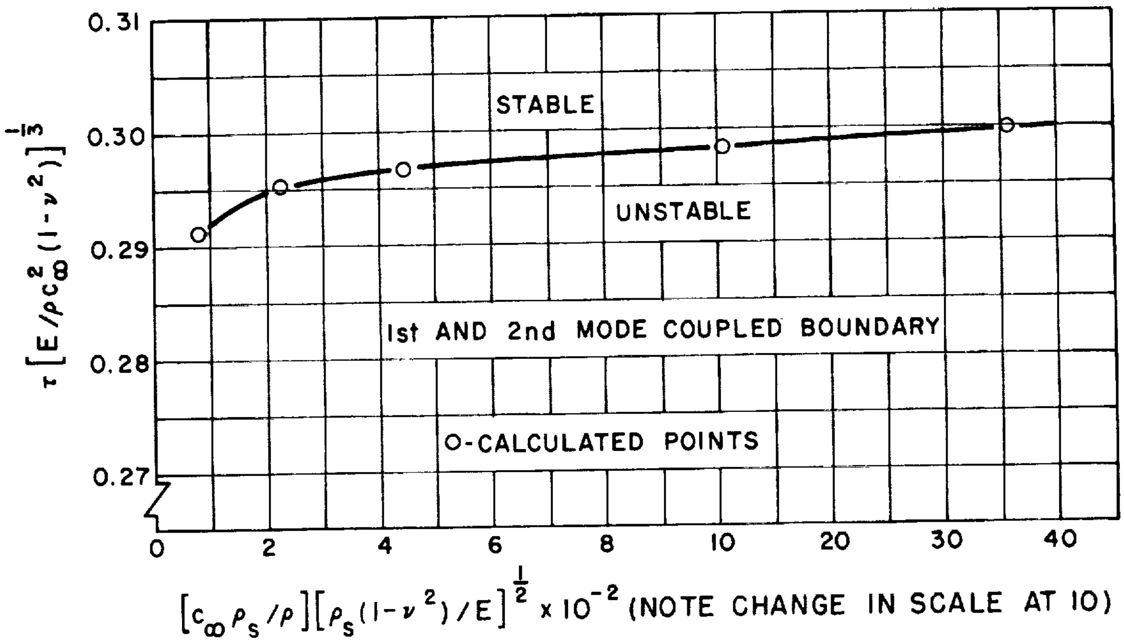

Fig. 24 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, $M = 1.5$, $s = 2$, $g = 0.01$ (Table IV)

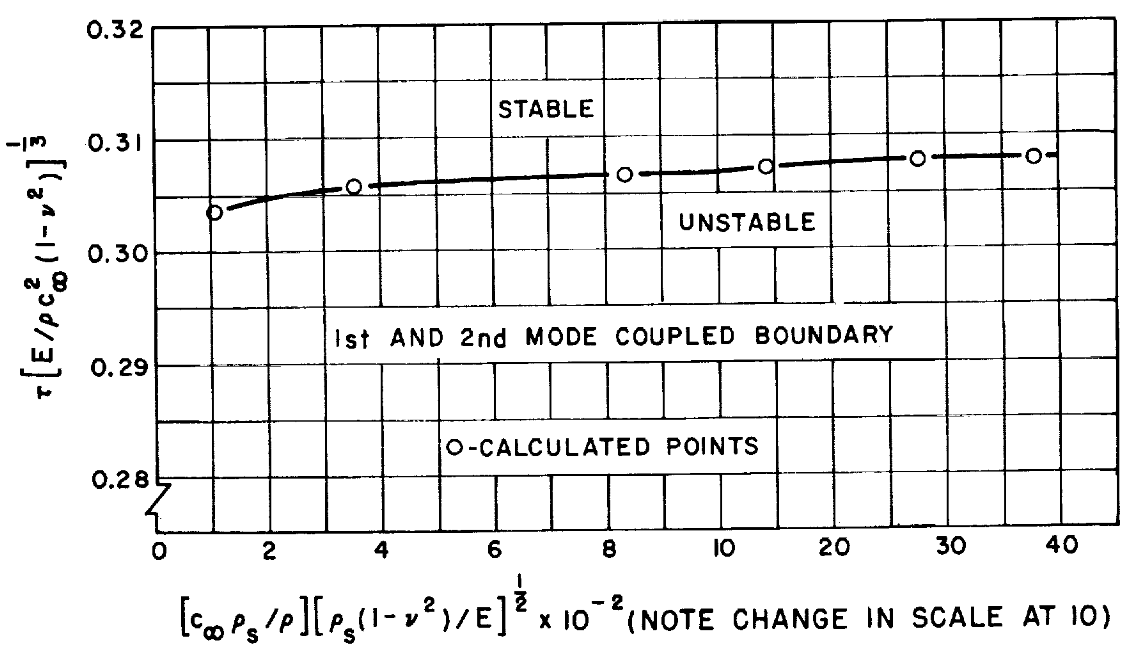

Fig. 25 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, $M = 2$, $s = 2$, $g = 0.01$ (Table IV)

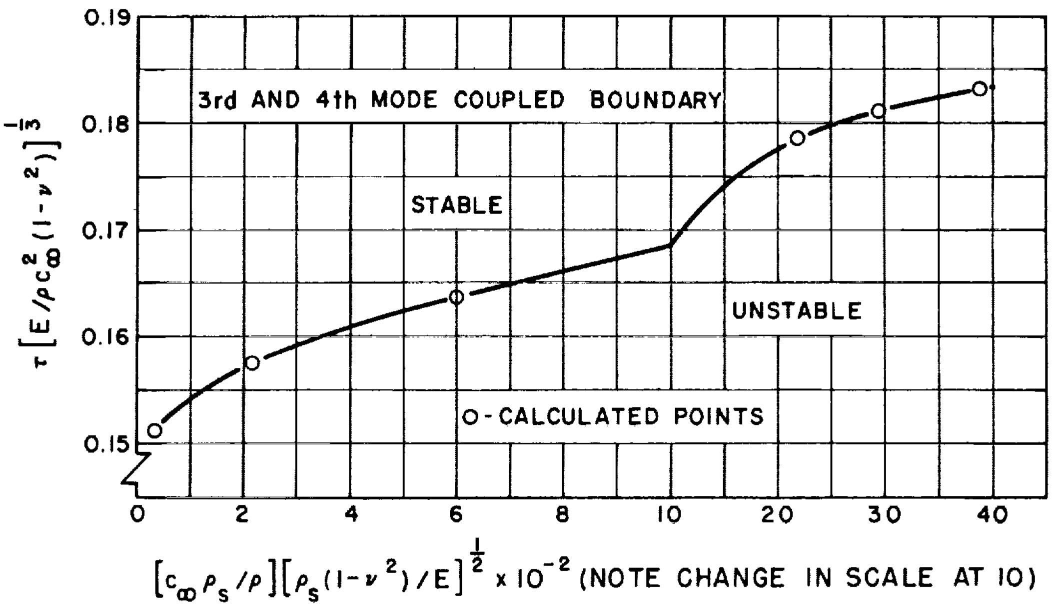

Fig. 26 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.35, s = 4, g = 0.01 (Table V)

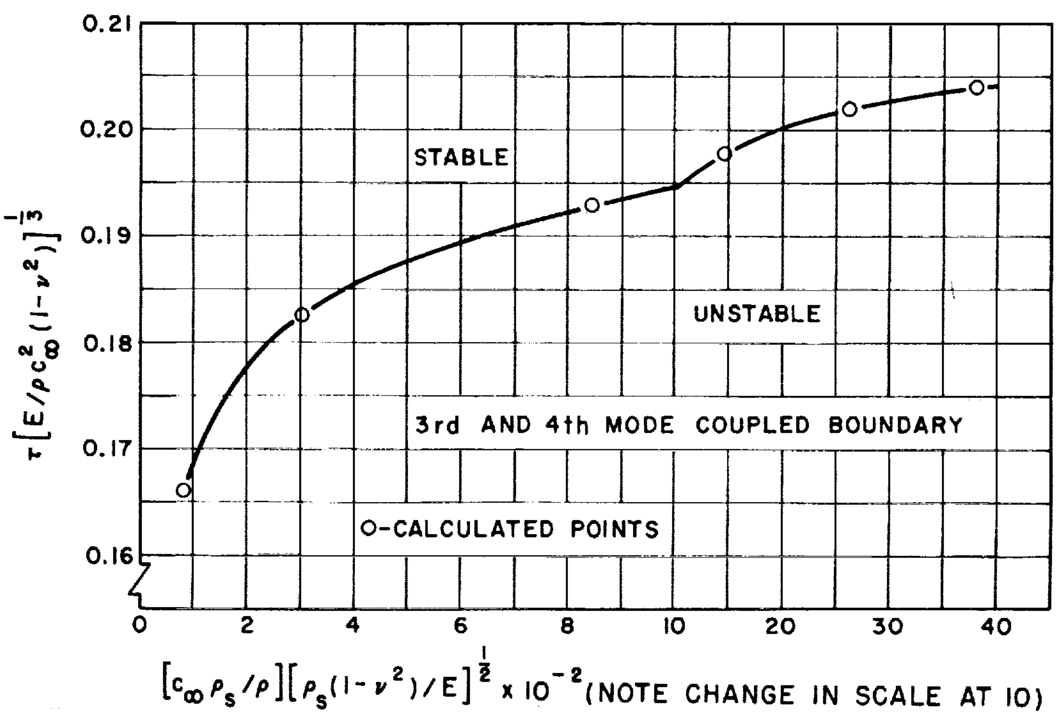

Fig. 27 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, $M = \sqrt{2}$, s = 4, g = 0.01 (Table V)

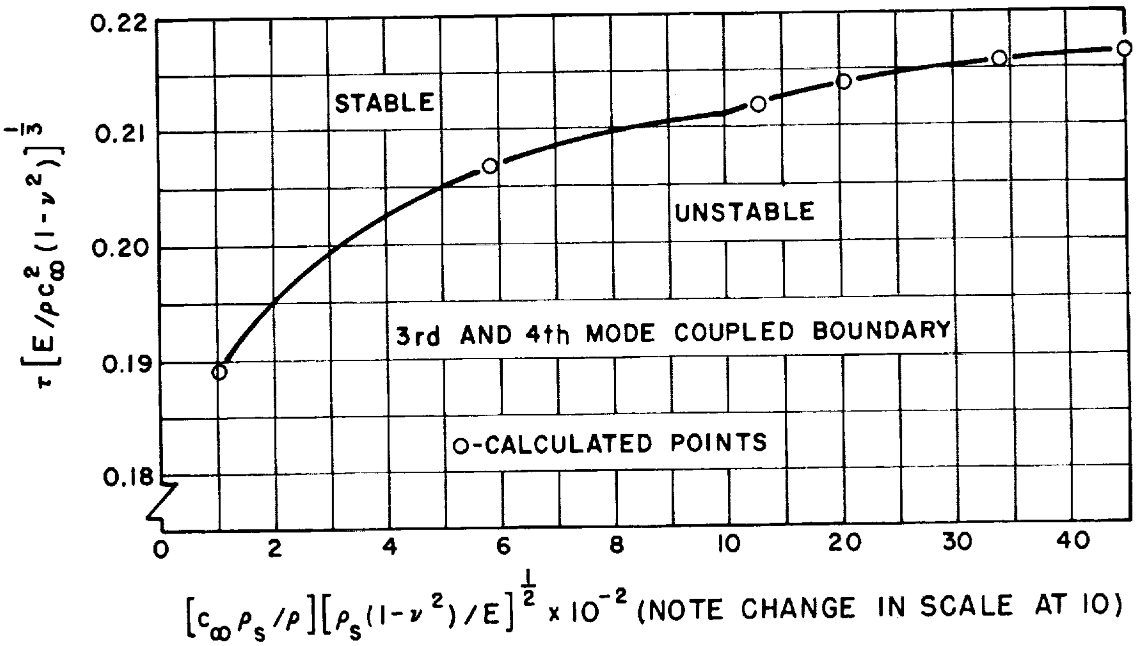

Fig. 28 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 1.5, s = 4, g = 0.01 (Table V)

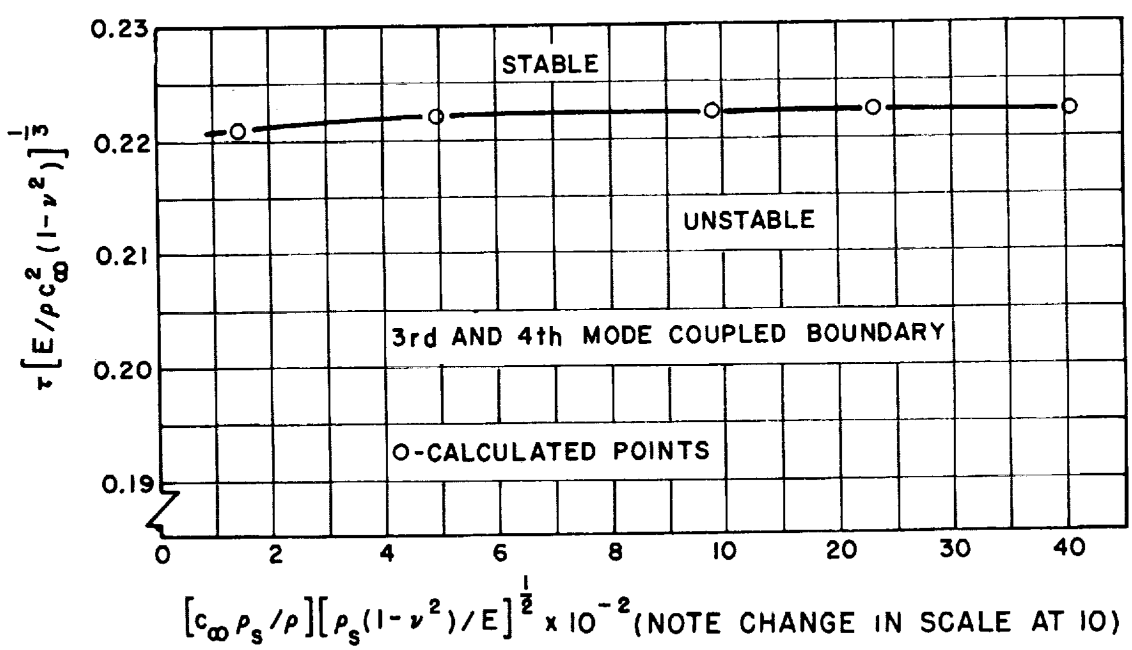

Fig. 29 - Critical Stability Boundary From a Four Mode Analysis for Simply Supported Flat Plates, M = 2, s = 4, g = 0.01 (Table V)

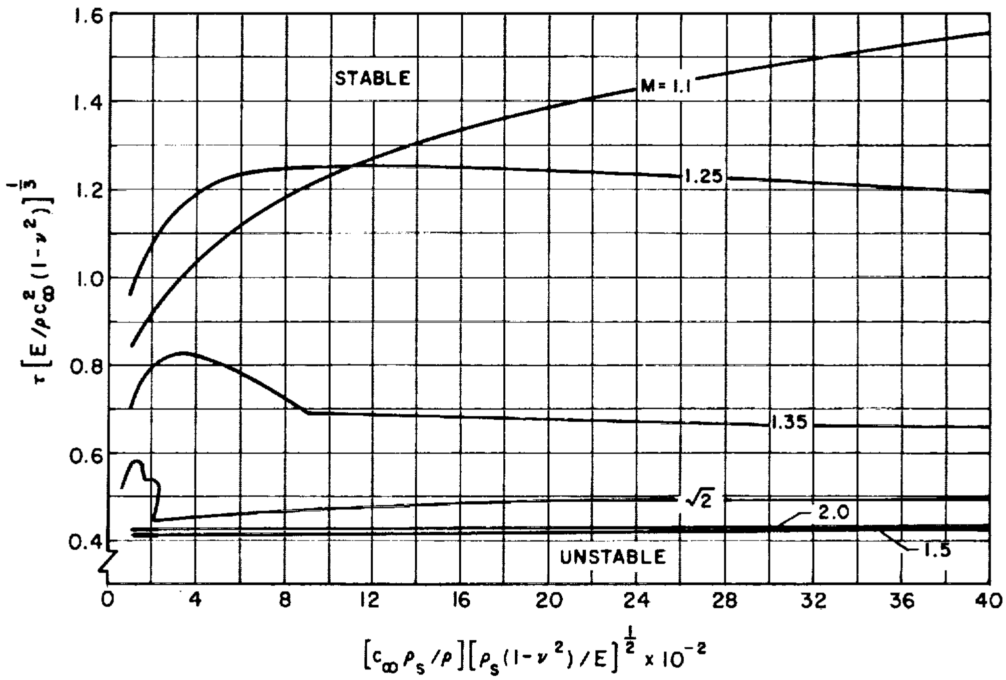

Fig. 30 - Variation of Critical Stability Boundaries With Mach Number for s = 1/4

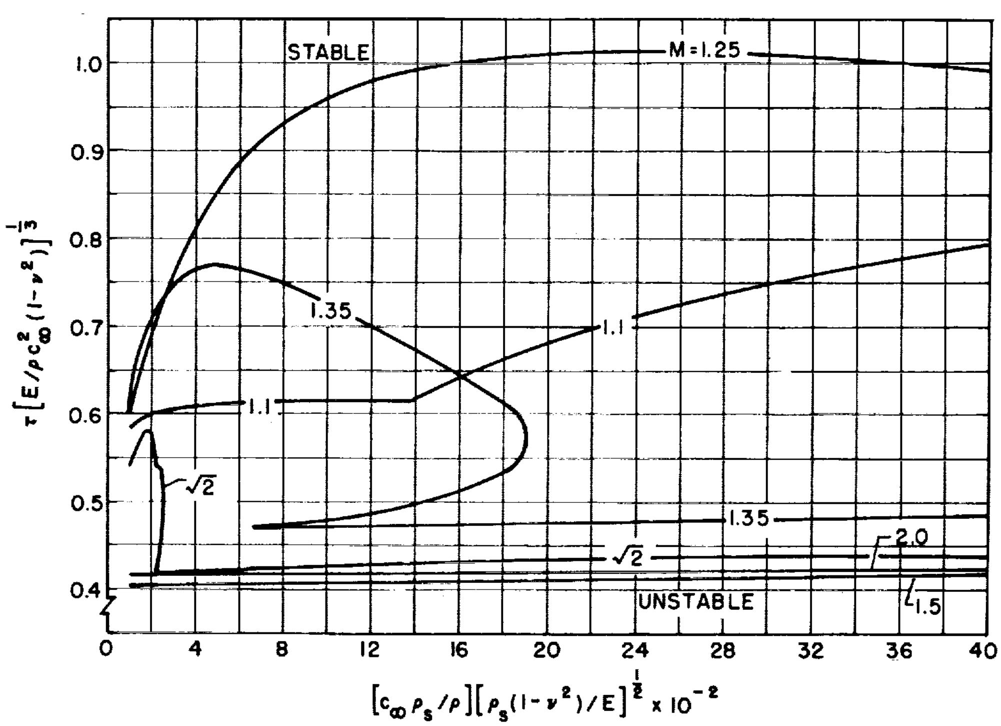

Fig. 31 - Variation of Critical Stability Boundaries With Mach Number for s = 1/2

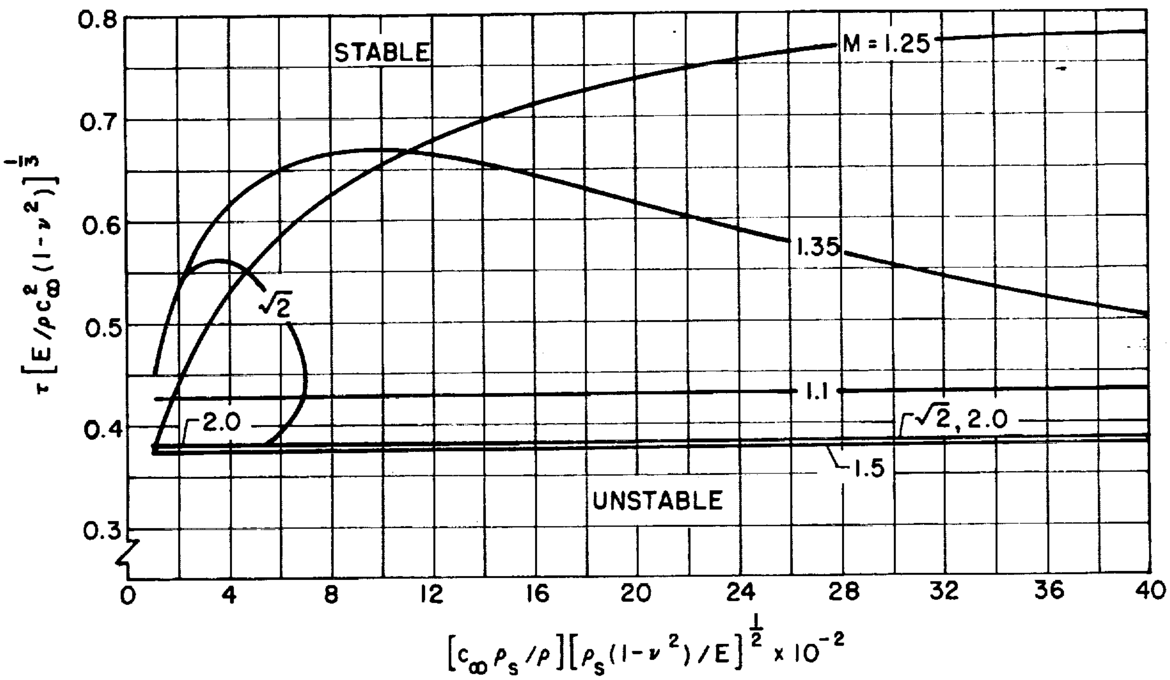

Fig. 32 - Variation of Critical Stability Boundaries With Mach Number for s = 1

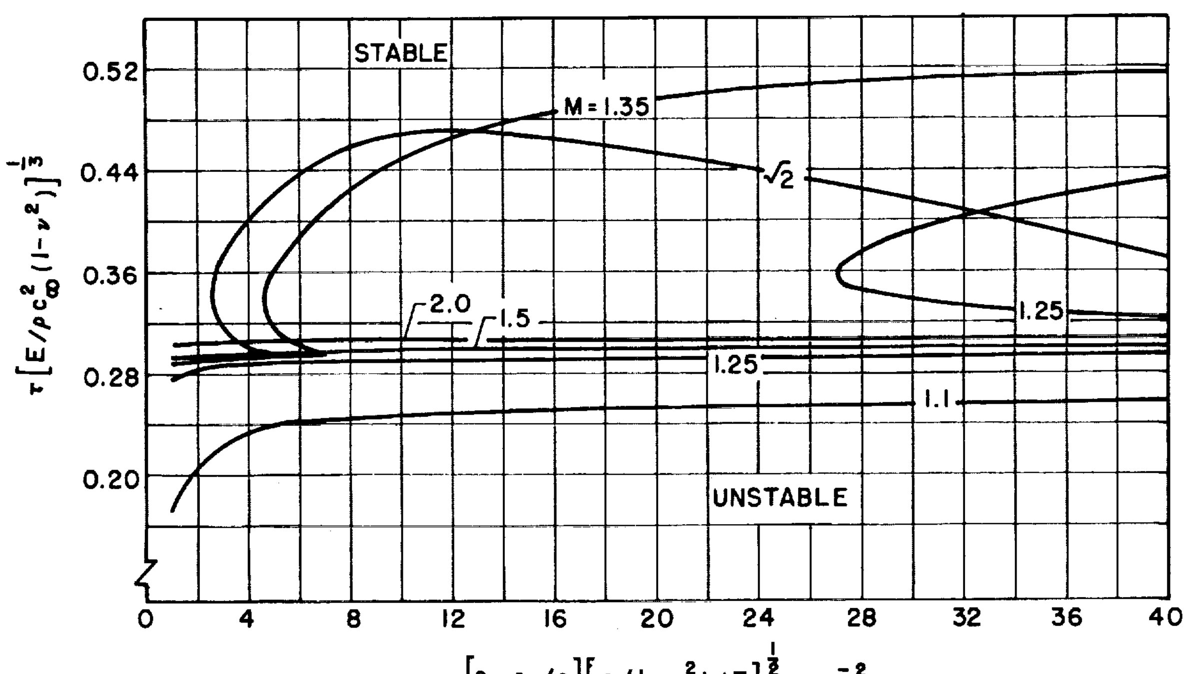

Fig. 33 - Variation of Critical Stability Boundaries With Mach Number for s = 2

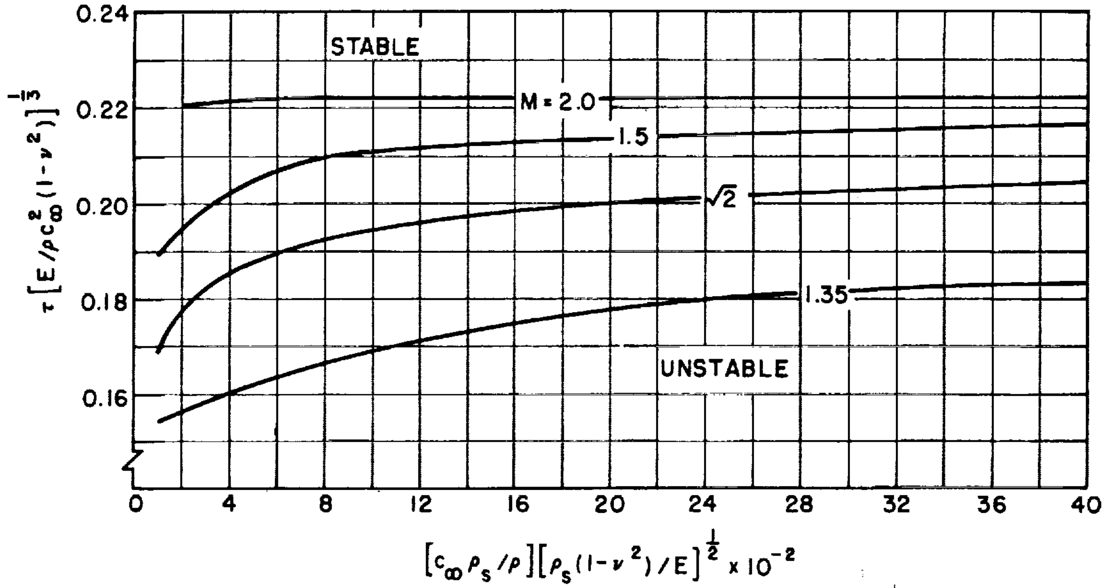

Fig. 34 - Variation of Critical Stability Boundaries With Mach Number for s = 4

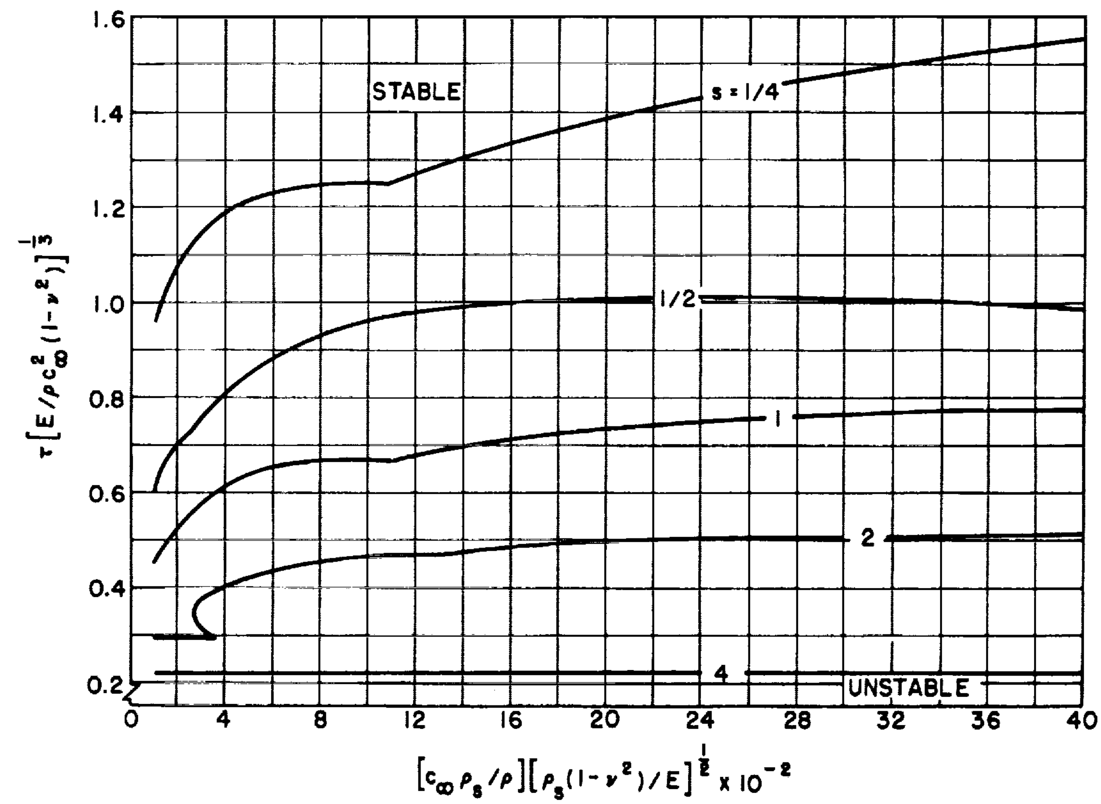

Fig. 35 - Variation of Critical Stability Boundaries With Length-Width Ratio s

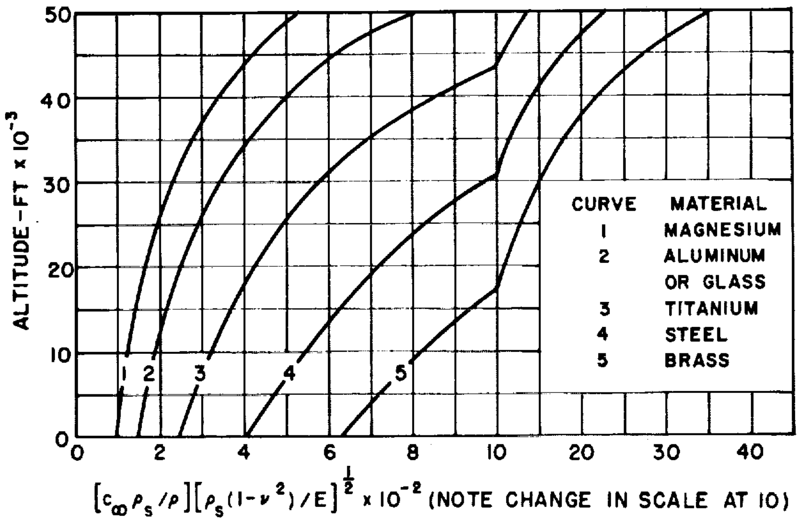

Fig. 36 - Variation of Parameter $[c_\infty \rho_s/\rho]\,[\rho_s(1-\nu^2)/E]^{1/2}$ With Material and Altitude

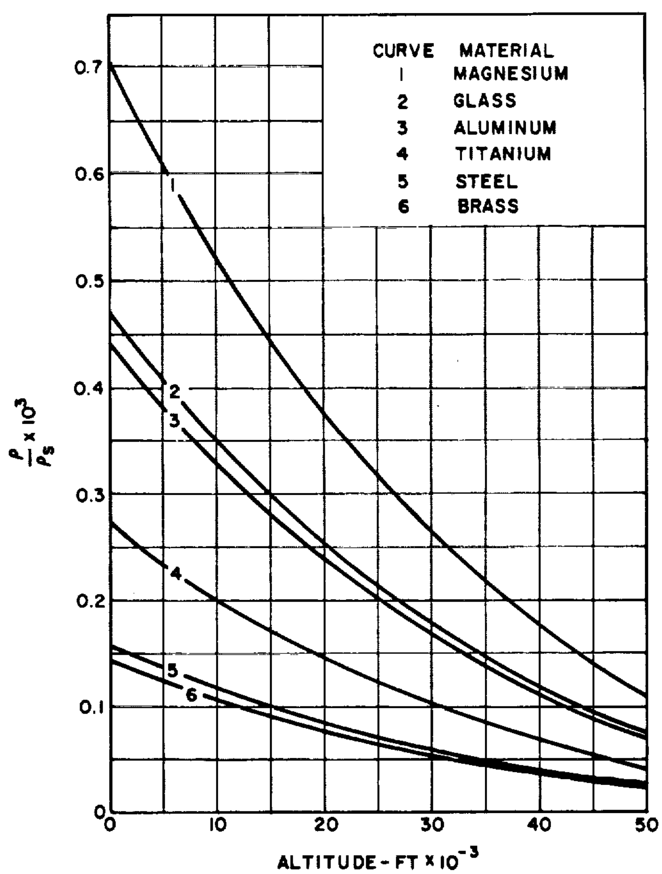

Fig. 37 - Variation of Density Ratio ρ/ρ_s With Panel Material and Altitude

APPENDIX B

TABLES I THROUGH V

TABLE I

FLUTTER FREQUENCY AND VECTOR FROM FOUR MODE ANALYSIS FOR SIMPLY SUPPORTED FLAT PLATES, s = 1/4

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.10	0.400	0.8152	59.84	1	0	0.1211	-3.5371	0.0071	-1.2757	0.0025	-3.6459
	0.332	1.0890	500.30	1	0	0.0511	-3.4769	0.0020	-1.3848	0.0010	-3.5223
	0.320	1.1654	740.94	1	0	0.0418	-3.4642	0.0016	-1.4006	0.0009	-3.5379
	0.300	1.3080	1402.10	1	0	0.0296	-3.4413	0.0010	-1.4095	0.0006	-3.4732
	0.280	1.4442	2466.28	1	0	0.0220	-3.4166	0.0007	-1.3867	0.0009	-4.0340
	0.260	1.5633	4053.14	1	0	0.0173	-3.3911	0.0005	-1.3374	0.0015	-4.3889
1.25	0.450	1.0594	164.06	1	0	0.0413	-3.2464	0.0009	-0.6248	0.0010	-3.2570
	0.400	1.1601	317.99	1	0	0.0313	-3.2301	0.0006	-0.6006	0.0007	-3.2820
	0.361	1.2121	500.30	1	0	0.0272	-3.2181	0.0005	-0.5594	0.0006	-3.2335
	0.330	1.2382	706.72	1	0	0.0254	-3.2090	0.0004	-0.5140	0.0006	-3.2225
	0.300	1.2500	974.29	1	0	0.0246	-3.2010	0.0004	-0.4642	0.0006	-3.2128
	0.260	1.2509	1509.96	1	0	0.0244	-3.1913	0.0003	-0.3848	0.0017	-3.5604
	0.220	1.2325	2396.33	1	0	0.0254	-3.1828	0.0003	-0.3129	0.0050	-3.4485
	0.200	1.2165	3072.62	1	0	0.0263	-3.1789	0.0003	-0.2651	0.0068	-3.4498
	0.180	1.1968	4021.31	1	0	0.0276	-3.1753	0.0003	-0.2081	0.0094	-3.4490
1.35	1.200	0.7743	160.77	0.0996	2.8711	1	0	0.0446	3.0733	0.0017	-0.3777
	1.000	0.8286	348.80	0.0827	-3.3468	1	0	0.0351	3.1283	0.0010	-0.1945
	0.800	0.7859	586.01	0.0970	3.0066	1	0	0.0397	3.1290	0.0011	-0.0778
	0.600	0.6872	928.90	0.1429	3.0659	1	0	0.0576	3.1340	0.0021	-0.0511
	0.200	0.6959	564.08	1	0	0.1329	3.1203	0.0061	-0.0656	0.0033	-3.1699
	0.150	0.6849	1323.39	1	0	0.1391	-3.1594	0.0067	-0.0507	0.0036	-3.1678
	0.120	0.6705	2534.40	1	0	0.1482	-3.1576	0.0075	-0.0424	0.0046	-3.1682
	0.100	0.6580	4313.67	1	0	0.1568	-3.1565	0.0082	-0.0372	0.0058	-3.1704

TABLE I (Continued)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
$\sqrt{2}$	2.300	0.5184	65.17	0.0221	-0.3585	0.1487	-3.4470	1	0	0.0830	-3.2248
	2.200	0.5734	103.26	0.0145	-0.2600	0.1115	-3.4302	1	0	0.0607	-3.2017
	2.100	0.5894	130.16	0.0127	-0.1581	0.1036	-3.3996	1	0	0.0549	-3.1836
	1.900	0.5772	166.56	0.0129	0.0228	0.1104	-3.3324	1	0	0.0562	-3.1595
	1.800	0.5555	174.90	0.0144	0.0896	0.1231	-3.2997	1	0	0.0620	-3.1532
	1.700	0.5246	174.83	0.0175	0.1300	0.1445	-3.2690	1	0	0.0724	-3.1501
	3.200	0.5238	143.01	0.0086	2.9724	0.0115	-0.2851	0.0845	2.7554	1	0
	3.000	0.5436	196.72	0.0075	3.0682	0.0098	-0.0365	0.0771	2.8344	1	0
	2.900	0.5397	213.94	0.0074	3.1091	0.0097	0.0794	0.0789	2.8744	1	0
	2.700	0.5134	228.17	0.0081	3.1704	0.0107	0.2821	0.0911	2.9502	1	0
	2.600	0.4927	226.36	0.0089	3.1882	0.0117	0.3616	0.1017	2.9844	1	0
	2.400	0.4397	202.70	0.0119	3.1930	0.0158	0.4371	0.1393	3.0415	1	0
	0.500	0.4338	75.30	1	0	0.6453	-3.1733	0.1210	-0.0596	0.0236	-3.1997
	0.300	0.4519	279.55	1	0	0.5340	-3.1621	0.0866	-0.0399	0.0180	-3.1769
	0.200	0.4697	790.33	1	0	0.4593	-3.1583	0.0657	-0.0232	0.0146	-3.1686
	0.160	0.4796	1419.31	1	0	0.4257	-3.1570	0.0570	-0.0308	0.0133	-3.1657
	0.140	0.4853	2026.49	1	0	0.4081	-3.1563	0.0526	-0.0296	0.0126	-3.1642
	0.120	0.4918	3071.75	1	0	0.3899	-3.1557	0.0483	-0.0284	0.0118	-3.1633
	0.110	0.4954	3892.78	1	0	0.3801	-3.1554	0.0459	-0.0278	0.0121	-3.1627
1.5	0.600	0.4142	60.42	1	0	0.8815	-3.2027	0.1966	-0.0881	0.0350	-3.2172
	0.400	0.4152	198.47	1	0	0.8471	-3.1827	0.1852	-0.0607	0.0332	-3.1929
	0.288	0.4173	500.30	1	0	0.8047	-3.1729	0.1715	-0.0478	0.0312	-3.1820
	0.240	0.4191	824.22	1	0	0.7782	-3.1689	0.1630	-0.0427	0.0299	-3.1778
	0.200	0.4215	1345.90	1	0	0.7474	-3.1656	0.1530	-0.0385	0.0285	-3.1744
	0.160	0.4255	2423.17	1	0	0.7060	-3.1652	0.1396	-0.0345	0.0266	-3.1710
	0.140	0.4284	3424.20	1	0	0.6796	-3.1690	0.1311	-0.0325	0.0252	-3.1695

TABLE I (Concluded)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
2.0	0.500	0.4281	62.01	1	0	0.9774	-3.2685	0.2259	-0.1571	0.0385	-3.2514
	0.300	0.4296	277.79	1	0	0.9482	-3.2206	0.2157	-0.1011	0.0371	-3.2143
	0.245	0.4302	500.30	1	0	0.9311	-3.2072	0.2103	-0.0855	0.0363	-3.2040
	0.200	0.4307	894.36	1	0	0.9150	-3.1966	0.2054	-0.0732	0.0357	-3.1959
	0.180	0.4311	1207.64	1	0	0.9046	-3.1918	0.2021	-0.0676	0.0353	-3.1922
	0.140	0.4322	2455.03	1	0	0.8762	-3.1822	0.1934	-0.0564	0.0342	-3.1851
	0.120	0.4332	3774.94	1	0	0.8564	-3.1774	0.1872	-0.0508	0.0334	-3.1645

TABLE II

FLUTTER FREQUENCY AND VECTOR FROM FOUR MODE ANALYSIS FOR SIMPLY SUPPORTED FLAT PLATES, s = 1/2

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.10	0.200	0.5980	174.89	1	0	0.2977	2.9457	0.0308	5.7958	0.0082	2.8087
	0.160	0.6067	399.35	1	0	0.2855	2.9824	0.0284	5.8829	0.0079	2.8757
	0.120	0.6137	1068.31	1	0	0.2761	3.0194	0.0265	5.9742	0.0077	2.9407
	0.100	0.6165	1937.81	1	0	0.2725	3.0380	0.0258	6.0212	0.0078	2.9689
	0.540	0.5324	689.19	0.3987	2.6261	1	0	0.1897	2.8257	0.0232	-0.9077
	0.540	0.6920	2122.80	0.1594	2.5026	1	0	0.0920	2.7531	0.0078	-1.2580
	0.520	0.7973	4019.41	0.0987	2.4888	1	0	0.0611	2.7590	0.0044	-1.3117
1.25	1.100	0.7343	241.12	0.1068	2.6875	1	0	0.0572	2.9357	0.0031	-0.8595
	1.030	0.8626	500.30	0.0673	2.7064	1	0	0.0354	2.9648	0.0016	-0.8430
	0.900	0.9745	1111.82	0.0483	2.7462	1	0	0.0241	3.0202	0.0009	-0.7101
	0.800	1.0088	1773.02	0.0447	2.7955	1	0	0.0212	3.0563	0.0008	-0.6791
	0.700	1.0125	269.92	0.0452	2.8472	1	0	0.0205	3.0848	0.0008	-0.5081
	0.600	0.9912	4025.08	0.0488	2.8981	1	0	0.0212	3.1076	0.0034	-0.5413
1.35	1.089	0.6531	142.90	0.1567	-3.3993	1	0	0.0722	3.0667	0.0372	-0.3473
	1.023	0.7301	248.30	0.1142	-3.3871	1	0	0.0515	3.0800	0.0210	-0.3251
	0.959	0.7562	339.00	0.1024	-3.3767	1	0	0.0459	3.0918	0.0017	-0.2790
	0.858	0.7680	500.30	0.0987	-3.3448	1	0	0.0432	3.1066	0.0015	-0.2024
	0.712	0.7462	807.40	0.1100	-3.2936	1	0	0.0459	3.1209	0.0015	-0.1110
	0.634	0.7202	1030.80	0.1204	-3.2749	1	0	0.0505	3.1257	0.0017	-0.0777
	0.600	0.7060	1144.62	0.1273	-3.2656	1	0	0.0534	3.1272	0.0019	-0.0669
	0.450	0.6161	1788.01	0.1903	-3.2221	1	0	0.0787	3.1301	0.0037	-0.0425
	0.390	0.5522	1941.39	0.2637	-3.2036	1	0	0.1081	3.1295	0.0068	-0.0418
	0.392	0.4938	1309.00	0.3716	-3.1954	1	0	0.1491	3.1275	0.0129	-0.0476
	0.438	0.4744	807.40	0.4236	-3.1978	1	0	0.1674	3.1254	0.0164	-0.0551

TABLE II (Continued)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.35	0.299	0.4589	248.30	1	0	0.4844	3.1018	0.0764	-0.0853	0.0172	3.0627
	0.181	0.4703	1030.80	1	0	0.4426	3.1131	0.0646	-0.0612	0.0151	3.0885
	0.130	0.4771	2650.34	1	0	0.4212	3.1179	0.0589	-0.0505	0.0145	3.0974
	0.110	0.4809	4261.01	1	0	0.4102	3.1197	0.0559	-0.0463	0.0145	3.0996
$\sqrt{2}$	0.400	0.4179	167.47	1	0	0.7033	-3.1834	0.1465	-0.0766	0.0292	-3.2197
	0.300	0.4215	375.31	1	0	0.6709	-3.1751	0.1353	-0.0621	0.0273	-3.2035
	0.200	0.4282	1141.52	1	0	0.6219	-3.1666	0.1189	-0.0490	0.0245	-3.1887
	0.150	0.4343	2478.75	1	0	0.5850	-3.1640	0.1068	-0.0427	0.0224	-3.1820
	0.140	0.4360	2983.40	1	0	0.5761	-3.1632	0.1039	-0.0414	0.0222	-3.1798
	0.130	0.4379	3633.68	1	0	0.5659	-3.1625	0.1006	-0.0402	0.0219	-3.1788
2	2.200	0.5298	85.23	0.0191	-0.3417	0.1362	-3.4476	1	0	0.0767	-3.2205
	1.900	0.5822	181.19	0.0123	-0.0519	0.1051	-3.3645	1	0	0.0550	-3.1690
	1.700	0.5505	214.79	0.0142	0.0935	0.1234	-3.3012	1	0	0.0629	-3.1535
	1.600	0.5222	219.99	0.0169	0.1335	0.1430	-3.2720	1	0	0.0726	-3.1503
	3.200	0.5030	130.17	0.0097	2.9386	0.0131	-0.3620	0.0933	2.7346	1	0
	3.000	0.5384	197.12	0.0077	3.0367	0.0101	-0.1124	0.0781	2.8090	1	0
	2.800	0.5334	237.45	0.0075	3.1178	0.0098	0.1162	0.0808	2.8874	1	0
	2.600	0.5046	251.84	0.0083	3.1726	0.0110	0.3073	0.0942	2.9602	1	0
	2.300	0.4309	224.43	0.0124	3.1873	0.0165	0.4365	0.1454	3.0462	1	0
1.5	0.500	0.4057	103.46	1	0	0.8616	-3.2035	0.2002	-0.0944	0.0377	-3.2291
	0.300	0.4076	452.37	1	0	0.8166	-3.1814	0.1846	-0.0628	0.0352	-3.1994
	0.250	0.4090	754.52	1	0	0.7959	-3.1762	0.1775	-0.0555	0.0341	-3.1927
	0.200	0.4111	1397.51	1	0	0.7674	-3.1712	0.1677	-0.0485	0.0325	-3.1862
	0.150	0.4149	3046.47	1	0	0.7257	-3.1663	0.1535	-0.0416	0.0302	-3.1796
	0.130	0.4174	4457.00	1	0	0.7025	-3.1643	0.1453	-0.0389	0.0295	-3.1768

TABLE II (Concluded)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
2.0	0.500	0.4174	65.68	1	0	0.9775	-3.2677	0.2376	-0.1584	0.0426	-3.2568
	0.300	0.4189	294.71	1	0	0.9491	-3.2201	0.2269	-0.1019	0.0409	-3.2176
	0.250	0.4195	500.30	1	0	0.9347	-3.2082	0.2219	-0.0878	0.0402	-3.2079
	0.200	0.4200	950.58	1	0	0.9171	-3.1964	0.2163	-0.0738	0.0394	-3.1983
	0.180	0.4204	1284.35	1	0	0.9071	-3.1916	0.2131	-0.0682	0.0390	-3.1945
	0.160	0.4208	1794.58	1	0	0.8949	-3.1869	0.2091	-0.0626	0.0385	-3.1907
	0.120	0.4224	4027.55	1	0	0.8610	-3.1774	0.1979	-0.0514	0.0369	-3.1828

TABLE III

FLUTTER FREQUENCY AND VECTOR FROM FOUR MODE ANALYSIS
FOR SIMPLY SUPPORTED FLAT PLATES, s = 1

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.10	0.200	0.4259	52.88	1	0	0.6471	-3.4463	0.1369	-0.6242	0.0246	-3.7650
	0.160	0.4261	181.30	1	0	0.6660	-3.3887	0.1434	-0.5023	0.0266	-3.6495
	0.133	0.4279	338.96	1	0	0.6592	-3.3495	0.1397	-0.4008	0.0263	-3.5427
	0.101	0.4298	807.29	1	0	0.6494	-3.3032	0.1396	-0.3315	0.0258	-3.4693
	0.080	0.4310	1597.24	1	0	0.6419	-3.2730	0.1315	-0.2709	0.0254	-3.4069
	0.060	0.4324	3570.47	1	0	0.6315	-3.2443	0.1270	-0.2133	0.0247	-3.3475
1.25	1.800	0.3019	27.93	0.2890	-0.4749	0.6511	-3.3768	1	0	0.4307	-3.4331
	1.600	0.5052	325.14	0.0235	-1.0017	0.1393	-3.7301	1	0	0.0988	-3.4332
	1.500	0.6272	808.86	0.0104	-0.9656	0.0746	-3.7378	1	0	0.0523	-3.3926
	1.400	0.6994	1411.79	0.0071	-0.8583	0.0559	-3.7051	1	0	0.0375	-3.3449
	1.300	0.7446	2154.12	0.0057	-0.7239	0.0481	-3.6605	1	0	0.0306	-3.2979
	1.200	0.7710	3061.45	0.0050	-0.5911	0.0446	-3.6107	1	0	0.0270	-3.2572
	1.150	0.7784	3588.76	0.0048	-0.5223	0.0439	-3.5848	1	0	0.0260	-3.2388
	1.100	0.7824	4174.00	0.0046	-0.4579	0.0437	-3.5587	1	0	0.0253	-3.2220
1.35	1.900	0.3864	65.36	0.0689	-0.5427	0.3118	-3.4733	1	0	0.1935	-3.2963
	1.750	0.6209	402.32	0.0091	-0.5423	0.0780	-3.5589	1	0	0.0482	-3.2594
	1.500	0.6678	809.77	0.0066	-0.2895	0.0653	-3.4748	1	0	0.0372	-3.1977
	1.250	0.6565	1338.71	0.0064	-0.0322	0.0697	-3.3859	1	0	0.0373	-3.1599
	1.000	0.6085	2090.19	0.0074	0.1534	0.0868	-3.3066	1	0	0.0448	-3.1437
	0.900	0.5817	2505.36	0.0083	0.1927	0.0986	-3.2792	1	0	0.0505	-3.1417
	0.700	0.5163	3714.86	0.0123	0.1843	0.1378	-3.2329	1	0	0.0682	-3.1428

TABLE III (Concluded)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
$\sqrt{2}$	1.900	0.4358	89.66	0.0367	-0.3835	0.2212	-3.4205	1	0	0.1303	-3.2279
	1.700	0.5593	281.29	0.0121	-0.2244	0.1080	-3.4143	1	0	0.0610	-3.1882
	1.350	0.5404	513.01	0.0123	0.0316	0.1202	-3.3181	1	0	0.0640	-3.1542
	1.150	0.4960	640.91	0.0165	0.0944	0.1533	-3.2690	1	0	0.0807	-3.1485
	1.000	0.4446	697.55	0.0262	0.0716	0.2083	-3.2361	1	0	0.1100	-3.1494
	1.000	0.3597	352.23	0.0804	-0.0403	0.3772	-3.2177	1	0	0.2086	-3.1594
	0.300	0.3806	480.90	1	0	0.7927	-3.2072	0.2066	-0.1163	0.0465	-3.2651
	0.250	0.3815	816.50	1	0	0.7819	-3.1980	0.2020	-0.1006	0.0456	-3.2478
	0.200	0.3828	1551.00	1	0	0.7673	-3.1890	0.1958	-0.0850	0.0443	-3.2308
	0.180	0.3835	2095.12	1	0	0.7599	-3.1855	0.1927	-0.0788	0.0437	-3.2240
	0.160	0.3851	2918.88	1	0	0.7510	-3.1819	0.1890	-0.0727	0.0429	-3.2172
	0.140	0.3856	4259.78	1	0	0.7393	-3.1784	0.1838	-0.0665	0.0424	-3.2110
1.5	0.600	0.3734	65.17	1	0	0.8837	-3.2593	0.2450	-0.1898	0.0540	-3.3361
	0.400	0.3749	219.14	1	0	0.8631	-3.2216	0.2353	-0.1306	0.0521	-3.2749
	0.300	0.3759	508.87	1	0	0.8464	-3.2038	0.2281	-0.1028	0.0507	-3.2462
	0.200	0.3779	1635.57	1	0	0.8179	-3.1866	0.2161	-0.0757	0.0483	-3.2182
	0.180	0.3785	2206.53	1	0	0.8091	-3.1832	0.2124	-0.0703	0.0477	-3.2129
	0.160	0.3793	3079.63	1	0	0.7989	-3.1798	0.2081	-0.0649	0.0469	-3.2074
	0.140	0.3803	4485.69	1	0	0.7868	-3.1764	0.2031	-0.0596	0.0458	-3.2017
2.0	0.500	0.3823	82.60	1	0	0.9819	-3.2673	0.2805	-0.1662	0.0594	-3.2772
	0.400	0.3831	160.10	1	0	0.9716	-3.2437	0.2752	-0.1364	0.0584	-3.2536
	0.300	0.3839	373.08	1	0	0.9561	-3.2201	0.2684	-0.1068	0.0570	-3.2304
	0.200	0.3850	1212.82	1	0	0.9284	-3.1967	0.2572	-0.0774	0.0550	-3.2074
	0.180	0.3854	1642.79	1	0	0.9197	-3.1921	0.2537	-0.0716	0.0544	-3.2030
	0.160	0.3858	2302.97	1	0	0.9093	-3.1874	0.2496	-0.0657	0.0537	-3.1985
	0.140	0.3864	3369.49	1	0	0.8961	-3.1827	0.2443	-0.0598	0.0529	-3.1942

TABLE IV

FLUTTER FREQUENCY AND VECTOR FROM FOUR MODE ANALYSIS FOR SIMPLY SUPPORTED FLAT PLATES, $s = 2$

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.10	0.230	0.2395	473.43	1	0	0.2404	1.8496	0.0510	-1.0484	0.1140	-4.3970
	0.210	0.2444	806.71	1	0	0.2585	2.0075	0.1009	-0.7189	0.1433	-4.3265
	0.200	0.2467	1052.10	1	0	0.2713	2.1317	0.1521	-0.5784	0.1715	-4.2946
	0.160	0.2539	2131.57	1	0	0.5654	1.3236	0.7682	3.4242	0.3099	-0.6632
	0.150	0.2546	2712.34	1	0	0.4955	1.3808	0.6357	3.4932	0.2400	-0.5433
	0.140	0.2552	3437.50	1	0	0.4499	1.4045	0.5554	3.5371	0.1979	-0.4442
	0.130	0.2557	4380.20	1	0	0.4160	1.4042	0.5024	3.5614	0.1702	-0.3629
1.25	0.500	0.2824	172.58	1	0	0.9090	-3.6542	0.3516	-0.8890	0.0934	-4.1058
	0.350	0.2891	545.06	1	0	0.9010	-3.5391	0.3251	-0.6889	0.0838	-3.8695
	0.300	0.2910	884.79	1	0	0.8986	-3.4942	0.3174	-0.6115	0.0814	-3.7828
	0.250	0.2928	1557.89	1	0	0.8957	-3.4461	0.3100	-0.5289	0.0791	-3.6926
	0.200	0.2943	3087.62	1	0	0.8912	-3.3950	0.3026	-0.4414	0.0769	-3.5996
	0.190	0.2946	3609.83	1	0	0.8900	-3.3845	0.3010	-0.4234	0.0764	-3.5806
	0.900	0.3266	3668.25	0.0262	2.8477	0.0734	-0.6083	0.3351	2.7958	1	0
	1.000	0.3375	3036.84	0.0220	2.8338	0.0614	-0.6995	0.2946	2.7439	1	0
	1.100	0.3556	2704.57	0.0175	2.8301	0.0486	-0.8005	0.2490	2.6878	1	0
	1.140	0.3672	2751.46	0.0150	2.8414	0.0414	-0.8488	0.2204	2.6599	1	0
	1.180	0.3854	2870.89	0.0126	2.8552	0.0343	-0.8995	0.1919	2.6314	1	0
	1.200	0.4440	4349.47	0.0077	2.9264	0.0196	-0.9564	0.1248	2.5914	1	0
1.35	0.650	0.2865	91.31	1	0	0.9497	-3.5230	0.3742	-0.6464	0.1104	-3.8411
	0.566	0.2894	142.90	1	0	0.9409	-3.4808	0.3617	-0.5755	0.1057	-3.7601
	0.379	0.2944	500.30	1	0	0.9241	-3.3802	0.3401	-0.4063	0.0982	-3.5732
	0.324	0.2956	807.40	1	0	0.9190	-3.3494	0.3345	-0.3546	0.0963	-3.5174
	1.229	0.2933	807.40	0.0624	3.2104	0.0921	-0.3449	0.4291	2.9078	1	0

TABLE IV (Continued)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.35	1.400	0.3015	602.83	0.0364	3.0776	0.0802	-0.3997	0.3884	2.8627	1	0
	1.600	0.3169	483.48	0.0302	3.0667	0.0619	-0.4949	0.3255	2.7977	1	0
	1.800	0.3498	479.78	0.0214	3.0703	0.0384	-0.6053	0.2334	2.7171	1	0
	1.877	0.4245	807.40	0.0156	3.1209	0.0167	-0.6306	0.1272	2.6553	1	0
	1.721	0.4897	1663.90	0.0061	3.1460	0.0090	-0.5159	0.0855	2.6853	1	0
	1.580	0.5092	2440.32	0.0065	3.2998	0.0081	-0.2992	0.0776	2.7378	1	0
	1.400	0.5154	3665.02	0.0060	3.3861	0.0073	-0.1212	0.0761	2.7990	1	0
	1.300	0.5123	4486.02	0.0060	3.4298	0.0072	-0.0325	0.0784	2.8327	1	0
$\sqrt{2}$	0.600	0.2912	125.42	1	0	0.9538	2.8539	0.3702	-0.4814	0.1116	2.6261
	0.340	0.2967	721.27	1	0	0.9296	2.9693	0.3445	-0.2902	0.1025	2.8336
	1.900	0.3274	290.01	0.0285	3.1452	0.0492	-0.3442	0.2850	2.8380	1	0
	1.800	0.3174	306.54	0.0314	3.1566	0.0568	-0.3061	0.3156	2.8669	1	0
	2.000	0.4505	715.87	0.0104	3.2229	0.0128	-0.2490	0.1075	2.7683	1	0
	1.800	0.4689	1120.25	0.0090	3.3153	0.0105	-0.0481	0.0971	2.8243	1	0
	1.500	0.4592	1830.92	0.0088	3.4322	0.0102	0.2034	0.1040	2.9084	1	0
	1.240	0.4294	2646.56	0.0097	3.4983	0.0116	0.3387	0.1261	2.9726	1	0
	1.000	0.3869	3632.03	0.0122	3.4953	0.0158	0.3183	0.1712	3.0212	1	0
	0.900	0.3628	4104.79	0.0143	3.4651	0.0201	0.2497	0.2042	3.0380	1	0
1.5	0.700	0.2913	79.80	1	0	0.9744	-3.4128	0.3868	-0.4439	0.1199	-3.6140
	0.500	0.2953	226.33	1	0	0.9568	-3.3402	0.3679	-0.3265	0.1128	-3.4872
	0.400	0.2968	446.51	1	0	0.9479	-3.3035	0.3596	-0.2671	0.1098	-3.4238
	0.300	0.2982	1062.52	1	0	0.9374	-3.2667	0.3511	-0.2074	0.1069	-3.3607
	0.200	0.2994	3564.98	1	0	0.9219	-3.2298	0.3403	-0.1475	0.1034	-3.2979
2.0	0.600	0.3036	104.27	0.9808	3.3066	1	0	0.3996	3.0639	0.1259	-0.0702
	0.400	0.3057	353.28	0.9965	3.2553	1	0	0.3917	3.0865	0.1224	-0.0506
	0.300	0.3066	833.14	1	0	0.9920	-3.2297	0.3848	-0.1321	0.1200	-3.2709
	0.250	0.3071	1431.43	1	0	0.9841	-3.2169	0.3796	-0.1138	0.1183	-3.2535

TABLE IV (Concluded)

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
2.0	0.200	0.3076	2765.87	1	0	0.9724	-3.2042	0.3725	-0.0955	0.1161	-3.2361
	0.180	0.3078	3779.10	1	0	0.9677	-3.1991	0.3695	-0.0882	0.1152	-3.2291

TABLE V

FLUTTER FREQUENCY AND VECTOR FROM FOUR MODE ANALYSIS FOR SIMPLY SUPPORTED FLAT PLATES, s = 4

M	k	$\alpha^{1/3}$	$(\mu^3/\alpha)^{1/2}$	q_1	θ_1	q_2	θ_2	q_3	θ_3	q_4	θ_4
1.35	0.600	0.1514	31.97	0.0612	-3.5627	0.2648	-4.9541	1	0	0.5755	-4.5126
	0.570	0.1575	214.75	0.0910	-3.6680	0.3099	-5.0716	1	0	0.6052	-4.4640
	0.530	0.1637	595.72	0.1648	-3.8482	0.3998	-5.3022	1	0	0.6535	-4.4155
	0.500	0.1786	2192.03	0.0275	-1.2889	0.4327	-1.7763	1	0	0.5736	-4.4202
	0.470	0.1812	2928.89	0.0450	-0.3557	0.3280	-1.9759	1	0	0.5913	-4.3320
	0.440	0.1833	3864.64	0.0587	-0.2689	0.2629	-2.1815	1	0	0.6116	-4.2651
$\sqrt{2}$	1.200	0.1661	83.13	0.9748	-0.4265	0.3219	-1.5764	1	0	0.7934	-4.5217
	1.000	0.1826	304.03	0.0840	0.2440	0.4436	-2.1968	1	0	0.6668	-4.1938
	0.800	0.1929	844.50	0.1107	-0.0822	0.3124	-2.7188	1	0	0.7142	-3.9582
	0.700	0.1976	1461.71	0.1074	-0.1413	0.2918	-2.9030	1	0	0.7301	-3.8543
	0.600	0.2017	2624.02	0.1029	-0.1685	0.2834	-3.0249	1	0	0.7402	-3.7561
	0.540	0.2039	3831.08	0.1004	-0.1740	0.2813	-3.0751	1	0	0.7447	-3.6984
1.5	1.500	0.1892	103.59	0.1438	0.1427	0.4437	-2.5301	1	0	0.7199	-4.0084
	1.000	0.2067	588.07	0.1372	-0.1289	0.3695	-3.0370	1	0	0.7294	-3.6692
	0.800	0.2118	1308.89	0.1298	-0.1399	0.3628	-3.1085	1	0	0.7215	-3.5585
	0.700	0.2138	2055.20	0.1269	-0.1354	0.3608	-3.1288	1	0	0.7166	-3.5071
	0.600	0.2156	3409.14	0.1245	-0.1262	0.3593	-3.1429	1	0	0.7135	-3.4564
	0.550	0.2164	4508.28	0.1234	-0.1200	0.3587	-3.1474	1	0	0.7113	-3.4321
2.0	1.500	0.2210	142.06	0.1649	-0.0674	0.5679	-3.1226	1	0	0.7029	-3.3282
	1.000	0.2221	496.62	0.1615	-0.0444	0.4658	-3.1304	1	0	0.6963	-3.2679
	0.800	0.2223	979.38	0.1605	-0.0344	0.4649	-3.1317	1	0	0.6958	-3.2463
	0.600	0.2224	2338.36	0.1598	-0.0243	0.4642	-3.1323	1	0	0.6965	-3.2259
	0.500	0.2224	4053.52	0.1594	-0.0191	0.4636	-3.1325	1	0	0.6979	-3.2160

NASA-Langley, 1963 D-2227